

Some Problems

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It strikes me, after having composed some lame problems—that still had some use I hope in regard to you all being able to write down and communicate your mathematical ideas—that we really have some good problems for which you should compose solutions. I hope each of you can try to write a clear and complete solution for the following problem by our next meeting on Tuesday September 13, 2022. This is inspired by Problem A1 on the 2021 Putnam exam and it is mentioned in my previous notes:

1 Minimal Route:

Given that a “move” from a point in the plane is executed by adding one of the twelve points

$$(\pm 5, 0), (\pm 4, 3), (\pm 3, 4), (0, \pm 5), (\pm 3, -3), (\pm 4, -4)$$

to the point in the plane which is the “current location,” what is the least number of moves required to reach the point

$$(2, 2)$$

if the “starting location” is $(0, 0)$?

2 Parity of Routes:

Starting at the origin $(0, 0)$ and with moves defined as above, show each point in the plane can be reached by either an odd number of moves or an even number of moves, but not both.

3 A Third Minimal Route Problem:

What is the least number of moves required to reach the point $(1, 0)$?