MATH 4802 **Name:** *Brandon Manville*
Problem A-2, 2021

1. For every positive number x, let

$$g(x) = \lim_{r\to 0}((x+1)^{r+1} - x^{r+1})^{1/r}$$

. Find

$$\lim_{x\to\infty} \frac{g(x)}{x}$$

.

$ln(g(x)) = \lim_{r\to 0} ln(((x+1)^{r+1} - x^{r+1})^{1/r})$

$= \lim_{r\to 0} \frac{ln((x+1)^{r+1} - x^{r+1})}{r}$

=$\frac{0}{0}$ L'H with respect to r

$= \lim_{r\to 0} \frac{ln(x+1)(x+1)^{r+1} - ln(x)x^{r+1}}{(x+1)^{r+1} - x^{r+1}}$

$= \frac{ln(x+1)(x+1) - ln(x)x}{(x+1)-x}$

$ln(g(x)) = ln(\frac{(x+1)^{x+1}}{x^x}) \Rightarrow *g(x) = \frac{(x+1)^{x+1}}{x^x}$

From here, there are two alternate solutions:

(1)(My Solution)

$\lim_{x\to\infty} \frac{\frac{(x+1)^{x+1}}{x^x}}{x}$

=$\lim_{x\to\infty} \frac{(x+1)^{x+1}}{x^{x+1}}$

$\to \lim_{x\to\infty} ln(\frac{(x+1)^{x+1}}{x^{x+1}})$

$= \lim_{x\to\infty}(x+1)ln(\frac{x+1}{x})$

$= \lim_{x\to\infty} \frac{ln(1+\frac{1}{x})}{\frac{1}{x+1}}$

$= \frac{0}{0}$ L'H with respect to x

$= \lim_{x\to\infty} \frac{\frac{\frac{-1}{x^2}}{1+\frac{1}{x}}}{\frac{-1}{(x+1)^2}} = \lim_{x\to\infty} \frac{(x+1)^2}{x^2+x}$=1

$\to \lim_{x\to\infty} \frac{g(x)}{x} = e$

(2) (Their Solution) Picking up from *,

$g(x) = \frac{(x+1)^{x+1}}{x^x} = (x+1)(x+1)^x x^{-x} = (x+1)((x+1)x^{-1})^x = (x+1)(1+\frac{1}{x})^x$

$= \lim_{x\to\infty} \frac{x+1}{x}(1+\frac{1}{x})^x = 1 \cdot e = e$