Exercise 1, Math 6514A

August 29, 2002

The beauty in mathematics is seeing the truth without effort.
—George Polya (1887-1985)

(a) Show that

$$u = \int_0^x \frac{1}{\sqrt{(1-\xi^2)(1-k^2\xi^2)}}\, d\xi = \int_0^\phi \frac{1}{\sqrt{1-k^2\sin^2\eta}}\, d\eta$$

where $x = \sin\phi$ and $k \in [0,1]$.

(b) Set $\mathrm{sn}(u) = x$. (The name of this function is read "ess-en.")

(c) Find a table of elliptic integrals (or use mathematica) to sketch the graph of $\mathrm{sn}(u)$. (Remember mathematica = Mathematica or Maple or Matlab or ...)

(d) Complete our analysis of the case $\dot\alpha_0 = 0$, $\alpha_0 + 2\pi k \in (0,\pi)$ to show that

$$\theta(t) = 2\arcsin\left[\sin\frac{\alpha_0}{2}\mathrm{sn}\left(\sqrt{\frac{g}{l}}(T/4-t)\right)\right]$$

where (as obtained in class)

$$T = 4\sqrt{\frac{l}{g}}K\left(\sin\frac{\alpha_0}{2}\right).$$

Most books give

$$\theta(t) = 2\arcsin\left[\sin\frac{\alpha_0}{2}\mathrm{sn}\left(\sqrt{\frac{g}{l}}t\right)\right].$$

What initial conditions does this satisfy?

(e) Do an aperiodic case (e.g., $\alpha_0 = \pi$, $\dot\alpha_0 \neq 0$).