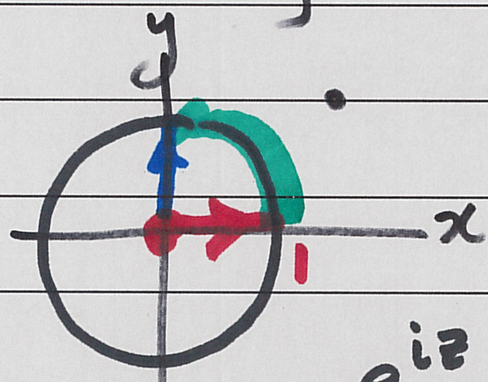
Lecture 6 MATH 6702 Jan 7, 2020

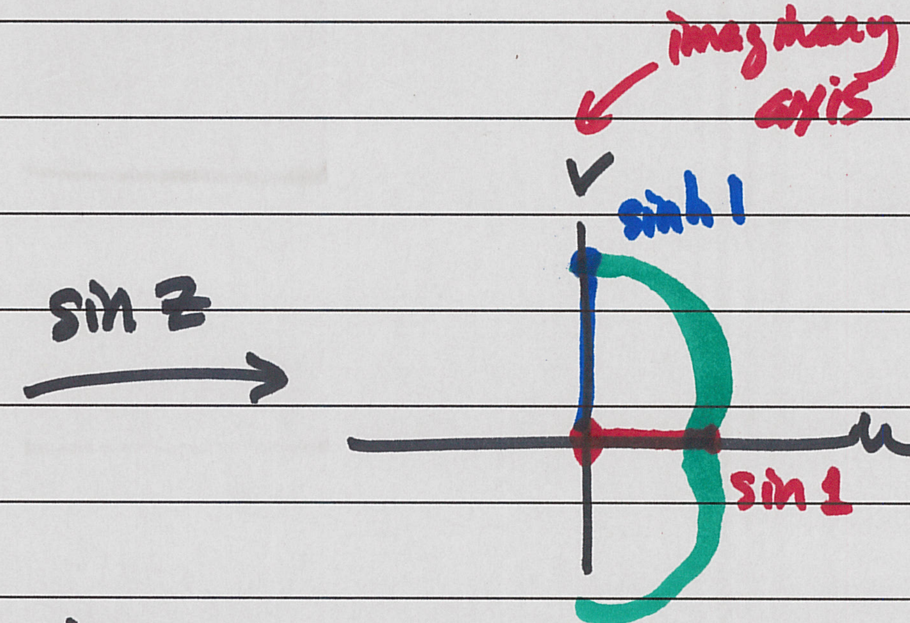
Homework #3

$\sin z = u + iv$

$\uparrow$

$z = x + iy$



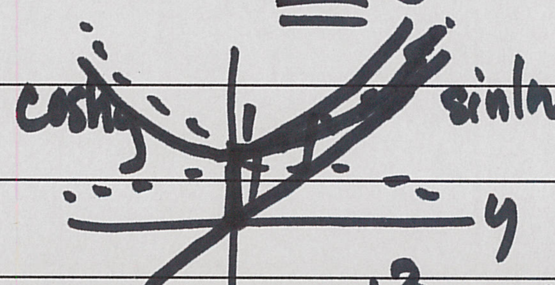


Hint: $\sin z = \dfrac{e^{iz} - e^{-iz}}{2i}$ (Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$)

$\rightsquigarrow \ = \sin x \underline{\underline{\cosh y}} + i \cos x \underline{\underline{\sinh y}}$

$\cosh y = \dfrac{e^{y} + e^{-y}}{2}$

$\sinh y = \dfrac{e^{y} - e^{-y}}{2}$



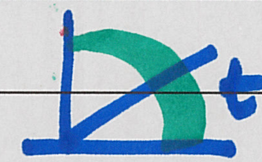
$\cosh^2 y - \sinh^2 y = 1$

[redacted]

To plot $\partial B_1(0)$ by

$u + iv = \sin x \cosh y + i \cos x \sinh y$

Use $\mathbb{r}(t) = (\underset{x}{\underset{\shortparallel}{\cos t}}, \underset{y}{\underset{\shortparallel}{\sin t}})$ $\quad 0 \le t \le \pi/2$



<u>Plot</u>: $(\sin(\cos t)\cosh(\sin t), \cos(\cos t)\sinh(\sin t))$

$0 \le t \le 2\pi$ for the entire curve

use math software

[redacted]

#3

continuity for $f: \mathbb{R} \to \mathbb{R}$

To show

For any $\underline{\underline{\varepsilon > 0}}$, there is some δ for which

$$|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon.$$

given

$$|f(\underset{x}{\overset{\shortparallel}{b}}) - f(\underset{x_0}{\overset{\shortparallel}{a}})| < \underbrace{c\,|b-a|^{\alpha}}_{? < \varepsilon ?}$$

Take $\delta = \left(\frac{\varepsilon}{c}\right)^{1/\alpha}$

$$\begin{matrix} |x - x_0| \\ < \delta \end{matrix} \Longrightarrow c\,|x - x_0|^{\alpha} < \varepsilon.$$

tell me what is δ

[redacted]

#5

$L : \mathbb{R}^n \to \mathbb{R}$ is determined by one <u>vector</u>

$\uparrow$

$\mathbb{R}^1 \to \mathbb{R}^1$

$L(x) = m x$

$\uparrow$

<u>one</u> number.

$L : \mathbb{R}^n \to \mathbb{R}$ $\leftarrow$ at $\mathbf{x}_0 \in \mathbb{R}^n$.

(gradient)

Guess : The vector we want is <u>$Du(\mathbf{x}_0)$</u>.

$$L(\mathbf{v}) = Du(\mathbf{x}_0) \cdot \mathbf{v}$$

<u>Zero</u> order approximation for $u(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}_0$?

Ans. The number $u(\mathbf{x}_0)$

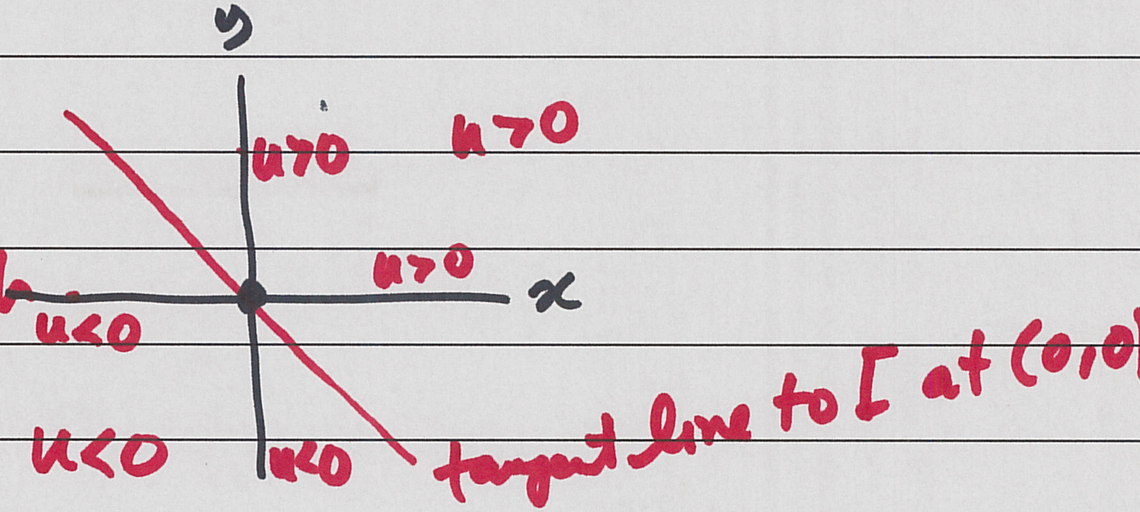
[redacted]

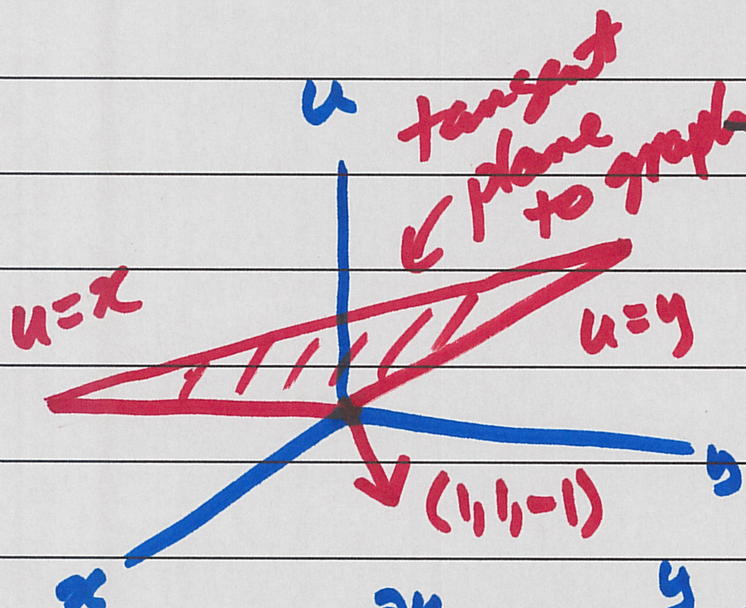
⑤

HW. 5 $\quad u(x,y) = xe^y + ye^x$

$\Gamma = \{ (x,y) \in \mathbb{R}^2 : u(x,y) = 0 \}.$

$(0,0) \in \Gamma$





tangent : $x+y-z=0$

$$\frac{\partial u}{\partial x} = e^y + ye^x \xrightarrow{(0,0)} 1$$

$$\frac{\partial u}{\partial y} = xe^y + e^x \xrightarrow{(0,0)} 1$$

[redacted]

6

$u(x,y) = xe^y + ye^x$

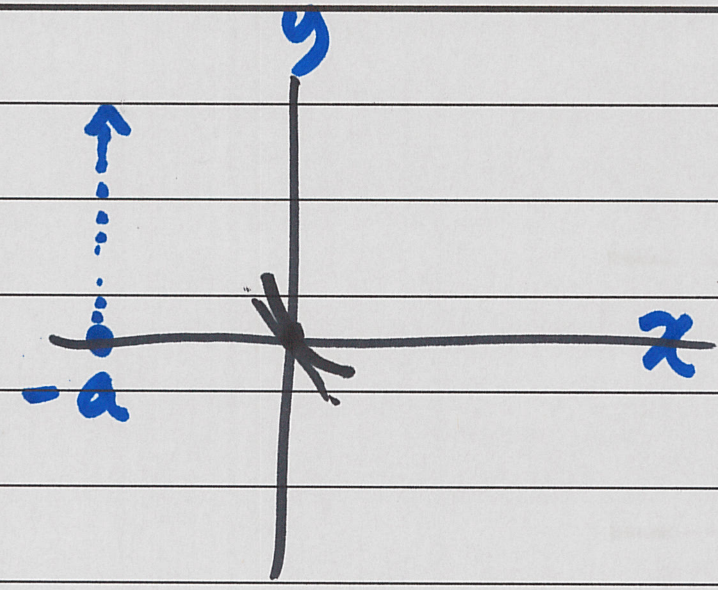
$\boxed{\text{If } \quad y = y(x)}$

$xe^y + ye^x = 0$

$e^y + xe^y y' + y'e^x + ye^x = 0$

$$y' = -\frac{ye^x + e^y}{xe^y + e^x}$$

$y'(0) = -1$



check $\quad y''(0) = 4 > 0.$

$\mathbb{r}(t) = (-a, 0) + t(0, 1)$
$\quad = (-a, t)$

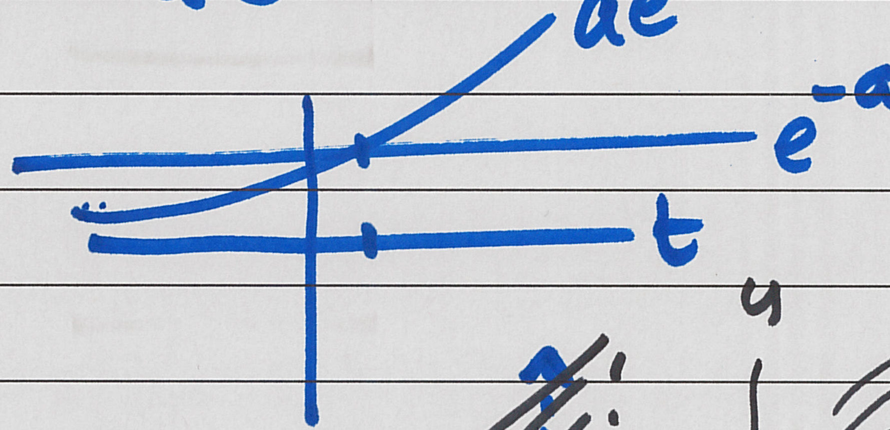
$u \circ \mathbb{r}(t) = -ae^t + te^{-a}$

$\frac{d}{dt}[u \circ \mathbb{r}(t)] = Du \circ \mathbb{r} \cdot \mathbb{r}' = \underline{\underline{-ae^t + e^{-a}}}$

[redacted]

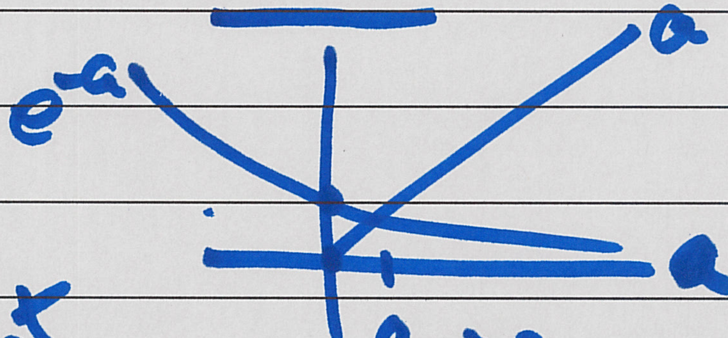
7.

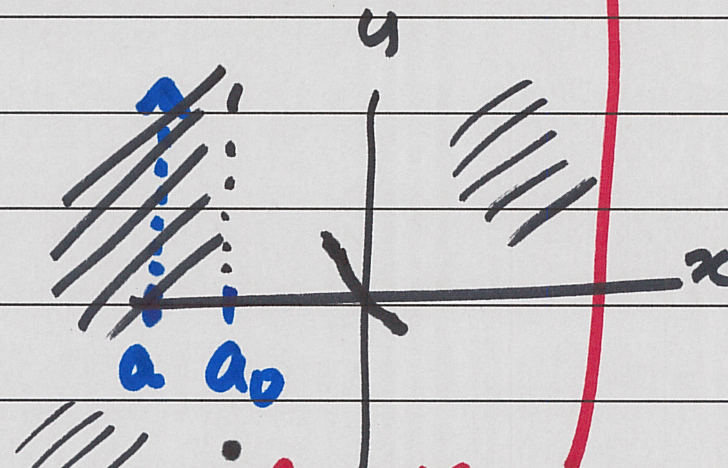
$$\frac{d}{dt}\left[u \circ r(t)\right] = -ae^{t} + e^{-a}$$



at $t = 0$

$-a + e^{-a}$





check for $a < a_0$

$[u \circ r(t)]' < 0.$

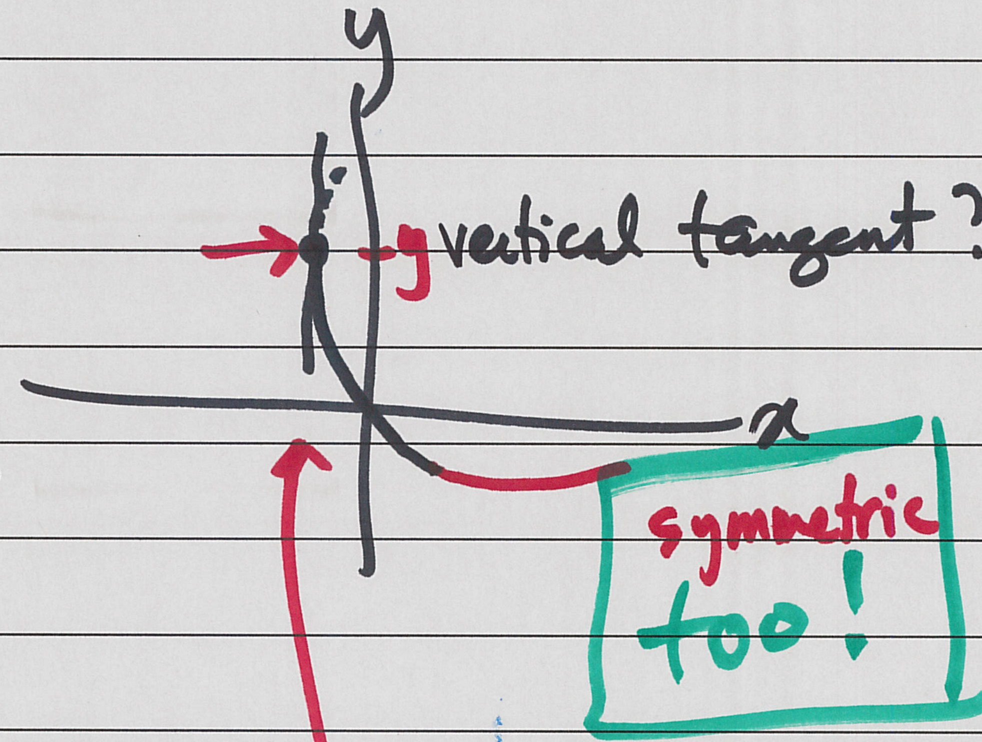
FindRoot

$$\boxed{-a_0 + e^{-a_0} = 0}$$

[redacted]

turning condition

$$\boxed{\frac{\partial u}{\partial y} = 0}$$



$$u(x,y) = xe^{y} + ye^{x}$$

$$\frac{\partial u}{\partial x} = e^{y} + ye^{x}$$

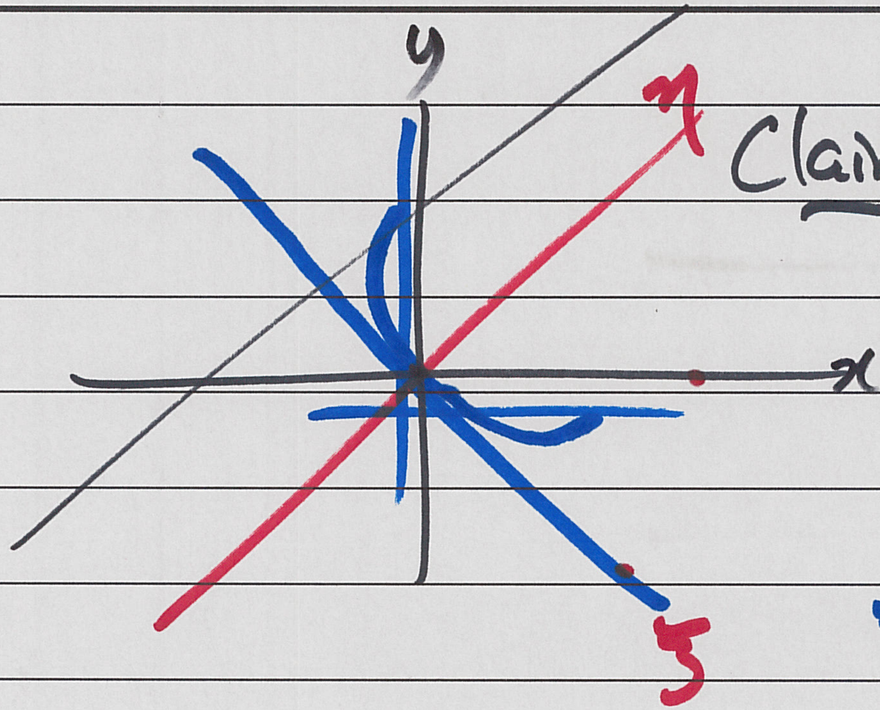
$$\frac{\partial u}{\partial y} = xe^{y} + e^{x}$$

turning $\begin{cases} xe^{y} + e^{x} = 0 \\ xe^{y} + ye^{x} = 0 \end{cases}$

Γ

$\Longrightarrow$ $y = 1$ $\boxed{ex + e^{x} = 0}$

[redacted]



<u>Claim</u>: There are no points on Γ where the tangent is parallel to $(1,1)$

$\Longrightarrow$ Γ is a graph over $y=-x$

$u(x,y) = xe^{y} + ye^{x}$

$M = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

clockwise rotation by $\pi/4$

$$u\left(M\begin{pmatrix}\xi\\ \eta\end{pmatrix}\right) = \frac{\sqrt{2}}{2}(\xi+\eta)e^{\frac{\sqrt{2}}{2}(-\xi+\eta)} + \frac{\sqrt{2}}{2}(-\xi+\eta)e^{\frac{\sqrt{2}}{2}(\xi+\eta)}$$

[redacted]

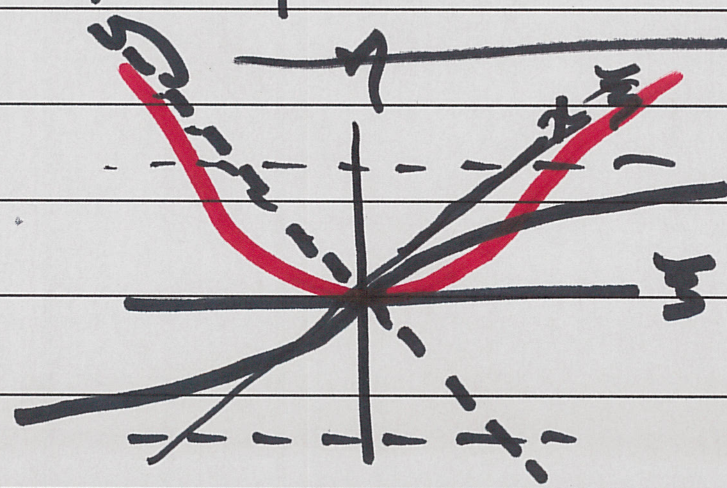
$$u\left(M\begin{pmatrix}\xi\\ \eta\end{pmatrix}\right) = \frac{\sqrt{2}}{2} e^{\frac{\sqrt{2}}{2}\eta} \left[(\xi+\eta) e^{-\frac{\sqrt{2}}{2}\xi} + (-\xi+\eta) e^{\frac{\sqrt{2}}{2}\xi} \right]$$

$$= \frac{\sqrt{2}}{2} e^{\frac{\sqrt{2}}{2}\eta} \left[-2\xi \sinh\left(\frac{\sqrt{2}}{2}\xi\right) + 2\eta \cosh\left(\frac{\sqrt{2}}{2}\xi\right) \right]$$

$$u\left(M\begin{pmatrix}\xi\\ \eta\end{pmatrix}\right) = 0$$

$$\boxed{u(x,y) = xe^{y} + ye^{x}}$$

$$\Longrightarrow \quad \eta = \xi \tanh\left(\frac{\sqrt{2}}{2}\xi\right)$$



$\tanh\left(\frac{\sqrt{2}}{2}\xi\right)$

odd

[redacted]

1-D Heat Equation

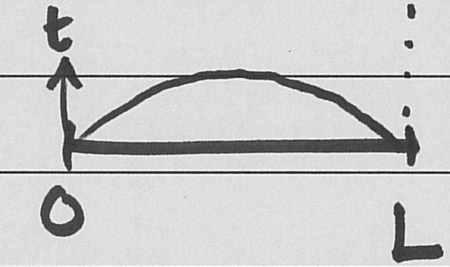
$u_t = \cancel{k}\, u_{xx}$

on $I \times [0,\infty) = [0,L] \times [0,\infty)$

$$\begin{cases} u(0,t) = 0 \\ u(L,t) = 0 \end{cases}$$

homogeneous boundary conditions

Models heat conduction on a thin rod.



$u(x,0) = g(x)$ initial condition

$$g = \sum_{j=1}^{\infty} g_j \sin \frac{j\pi x}{L}$$

Fourier basis.

[redacted]