Fourier Series

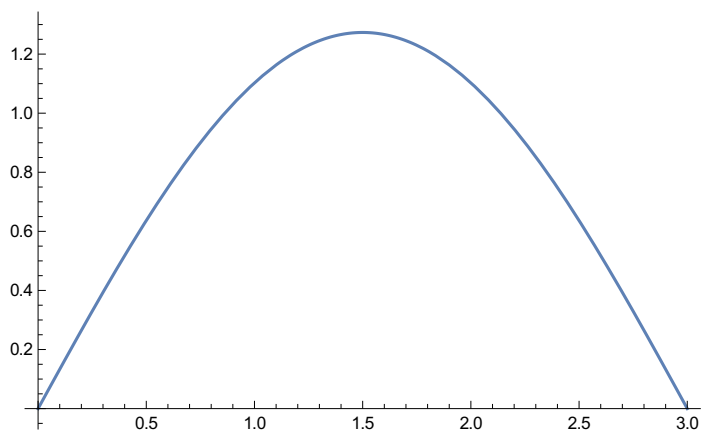## Representing a constant function

```
fk[x_, k_, a_] := Sum[4 Sin[(2 j + 1) Pi x / a] / ((2 j + 1) Pi), {j, 0, k}]
```
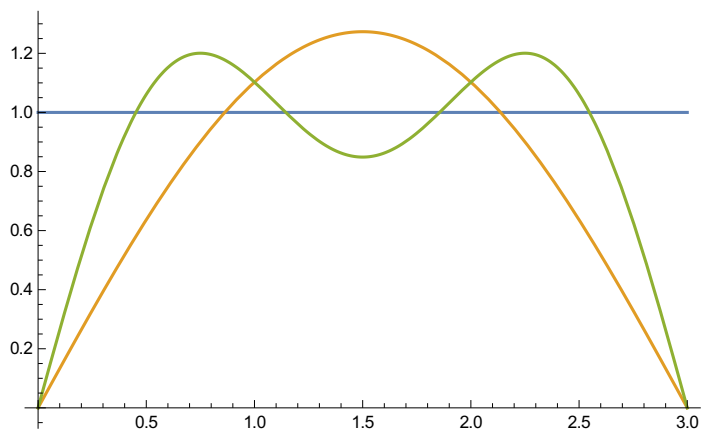
```
fk[x, 2, 3]
```

$$\frac{4 \, \text{Sin}\left[\frac{\pi x}{3}\right]}{\pi} + \frac{4 \, \text{Sin}[\pi x]}{3 \pi} + \frac{4 \, \text{Sin}\left[\frac{5 \pi x}{3}\right]}{5 \pi}$$

```
Plot[fk[x, 0, 3], {x, 0, 3}]
```
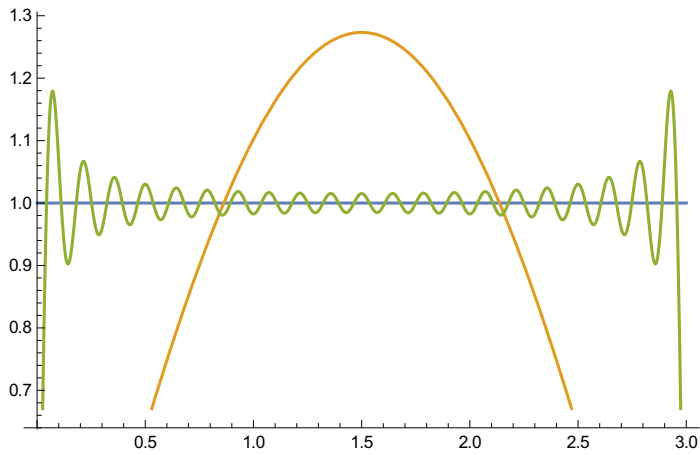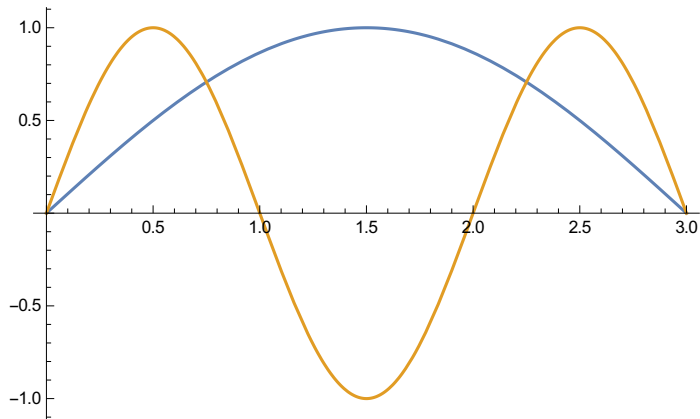


```
Plot[{1, fk[x, 0, 3], fk[x, 1, 3]}, {x, 0, 3}]
```

$\text{Plot}\Big[\{1, \text{fk}[x, 0, 3], \text{fk}[x, 20, 3]\}, \{x, 0, 3\}\Big]$

$b[x\_, j\_, a\_] = \text{Sin}\Big[(2\,j + 1)\,\text{Pi}\,x / a\Big]$

$\text{Sin}\Big[\dfrac{(1 + 2\,j)\,\pi\,x}{a}\Big]$

$\text{Plot}\Big[\{b[x, 0, 3], b[x, 1, 3]\}, \{x, 0, 3\}\Big]$

$\text{Plot}\Big[\{1, \text{fk}[x, 100, 3]\}, \{x, 0, 3\},$
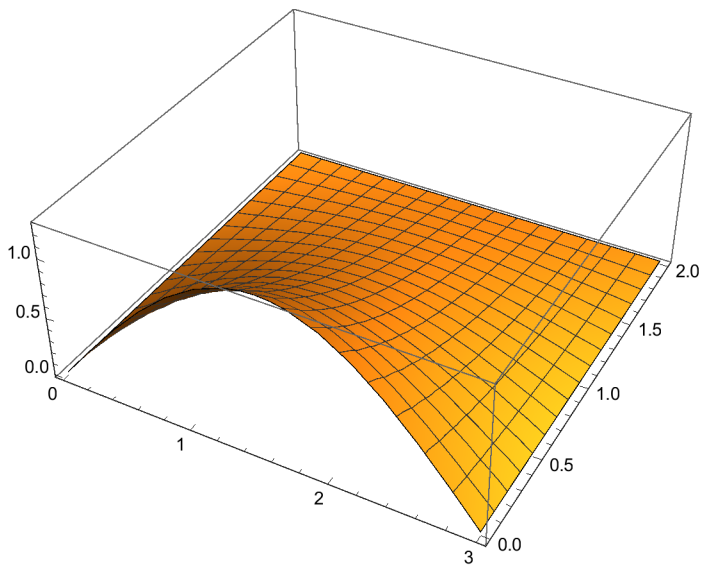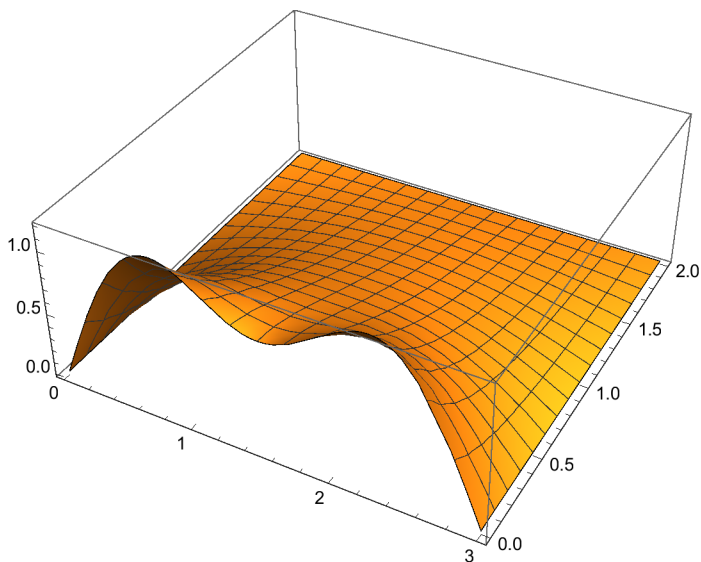$\text{PlotStyle} \to \{\{\text{Red}\}, \{\text{Thick}, \text{Black}\}\}, \text{PlotRange} \to \{0, 1.3\}\Big]$
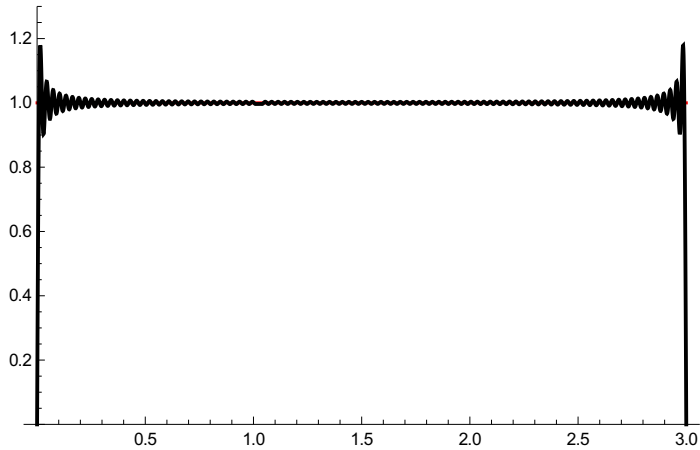
13.2.11

```
A[x_, j_, a_] = b[x, j, a]
B[y_, j_, a_, b_] = Cosh[(2 j + 1) Pi y / a] - Coth[(2 j + 1) Pi b / a] Sinh[(2 j + 1) Pi y / a]
```

$$\text{Sin}\left[\frac{(1 + 2 j) \pi x}{a}\right]$$

$$\text{Cosh}\left[\frac{(1 + 2 j) \pi y}{a}\right] - \text{Coth}\left[\frac{b (1 + 2 j) \pi}{a}\right] \text{Sinh}\left[\frac{(1 + 2 j) \pi y}{a}\right]$$

```
u[x_, y_, k_, a_, b_] := Sum[4 A[x, j, a] B[y, j, a, b] / ((2 j + 1) Pi), {j, 0, k}]
```
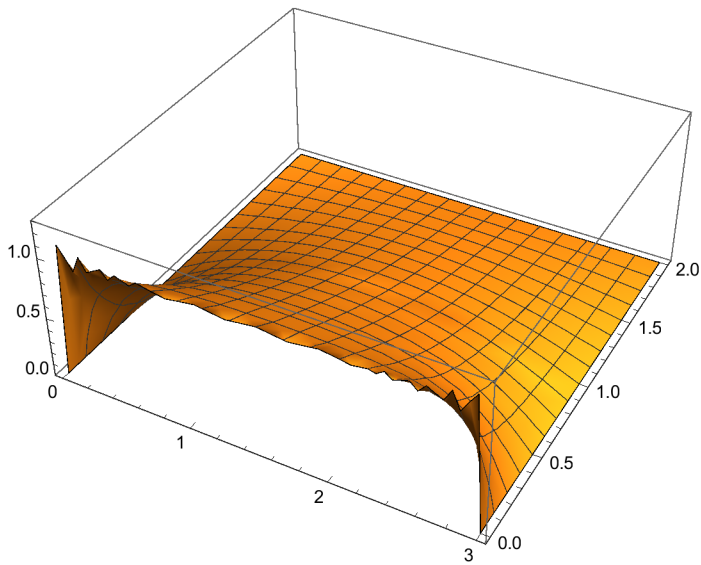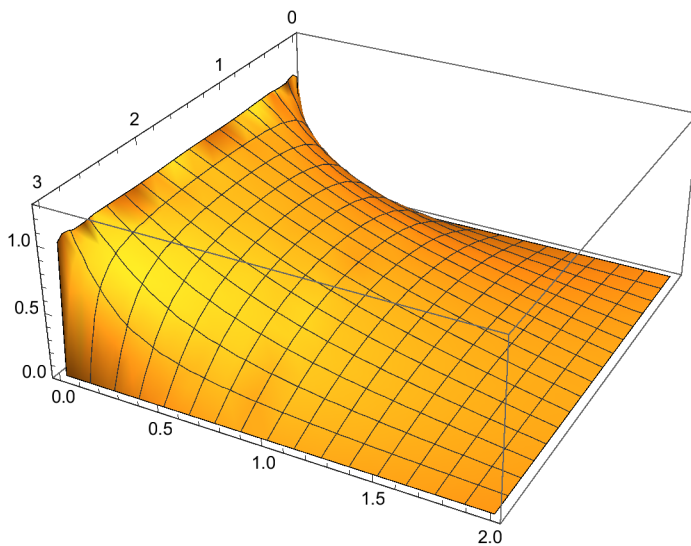
```
Plot3D[u[x, y, 0, 3, 2], {x, 0, 3}, {y, 0, 2}]
```



```
Plot3D[u[x, y, 1, 3, 2], {x, 0, 3}, {y, 0, 2}]
```

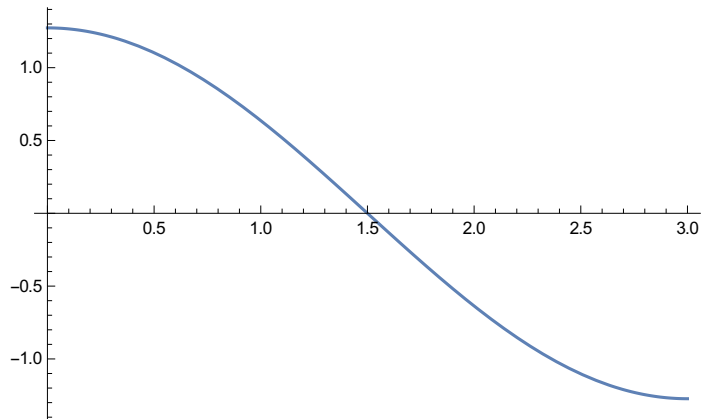Plot3D$\big[$u[x, y, 20, 3, 2], {x, 0, 3}, {y, 0, 2}, PlotRange $\to$ {0, 1.3}$\big]$

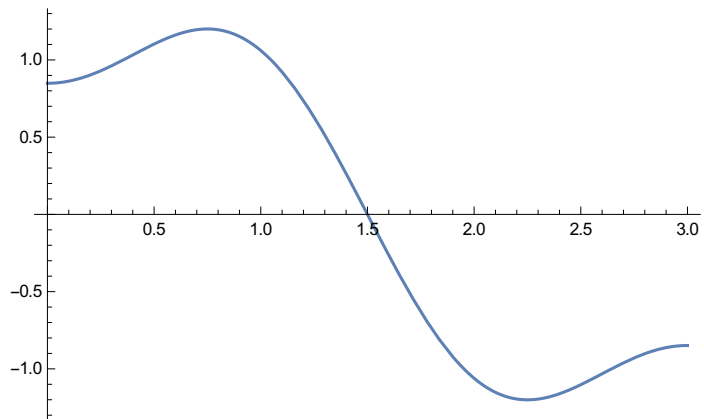Plot3D$\big[$u[x, y, 100, 3, 2], {x, 0, 3}, {y, 0, 2}, PlotRange $\to$ {0, 1.3}$\big]$

13.3.3

f$\big[$x_, k_, L_$\big]$:= Sum$\big[$4 Sin$\big[$j Pi / 2$\big]$ Cos$\big[$j Pi x / L$\big]$ / (j Pi), {j, 1, k}$\big]$
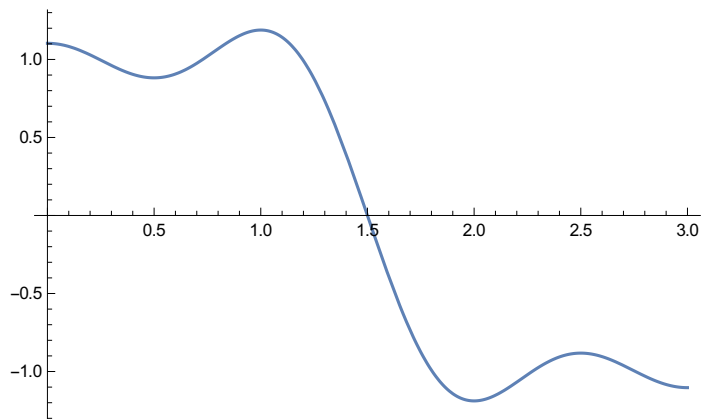
```
Plot[f[x, 1, 3], {x, 0, 3}]
```
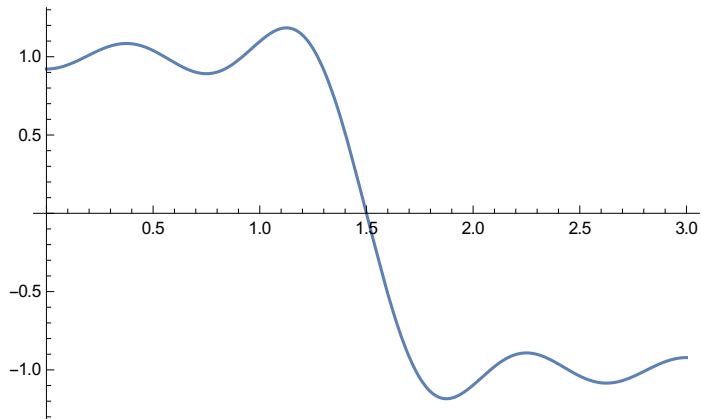


```
Plot[f[x, 3, 3], {x, 0, 3}]
```
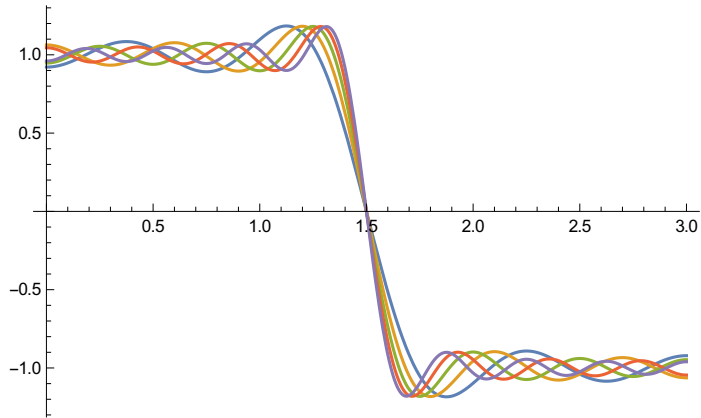


```
Plot[f[x, 5, 3], {x, 0, 3}]
```
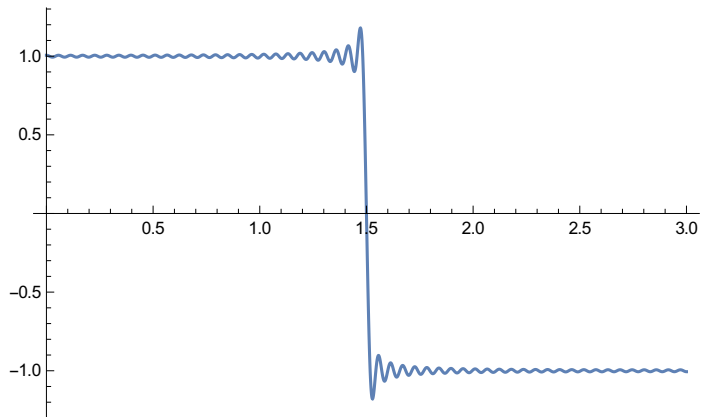
```
Plot[f[x, 7, 3], {x, 0, 3}]
```



```
Plot[{f[x, 7, 3], f[x, 9, 3], f[x, 11, 3], f[x, 13, 3], f[x, 15, 3]}, {x, 0, 3}]
```
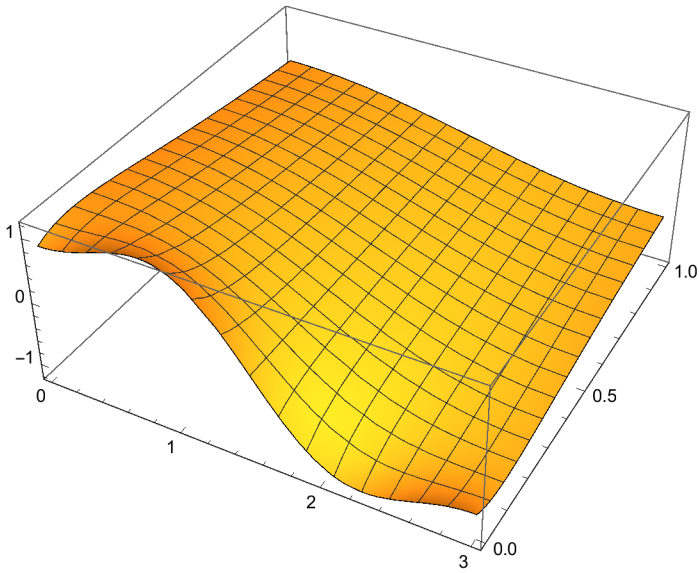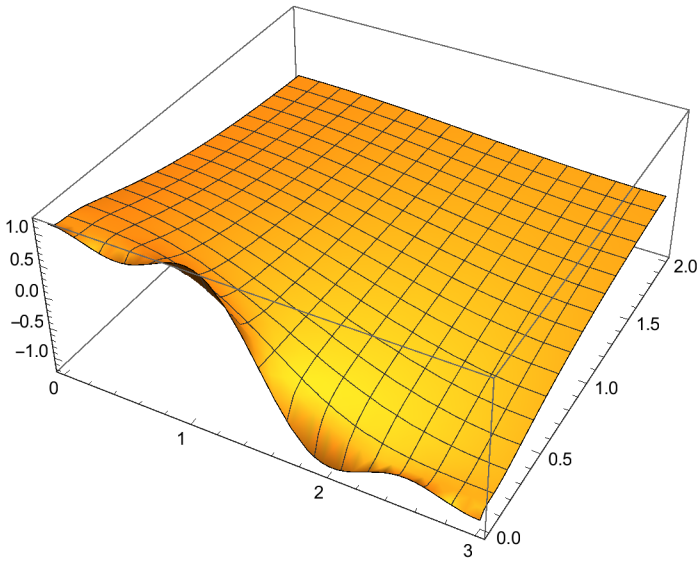


```
Plot[f[x, 105, 3], {x, 0, 3}]
```



```
u[x_, t_, k_, L_] :=
  Sum[4 Sin[j Pi / 2] Cos[j Pi x / L] E ^ (-j ^ 2 Pi ^ 2 t / L ^ 2) / (j Pi), {j, 1, k}]
```

`Plot3D[u[x, t, 3, 3], {x, 0, 3}, {t, 0, 1}]`

`Plot3D[u[x, t, 5, 3], {x, 0, 3}, {t, 0, 2}]`

```
Plot3D[u[x, t, 105, 3], {x, 0, 3}, {t, 0, 2}]
```