Math 6702, Assignment 1

Introduction

1. (Exercise 1) If $f(x) = e^x$ and $y = f(x)$, then $y'' = e^x$ and $y'' = y$. Find the solutions of these two ODEs.

   Problems 2 and 3 involve the following ordinary differential equations:

$$y'' = \sin(x^2) \tag{1}$$
$$y'' = e^{-x^2}. \tag{2}$$

2. (Exercise 2) Use the following functions to solve (1) and (2)

$$f_1(x) = \int_0^x \int_0^\xi \sin(t^2)\, dt d\xi.$$

$$f_2(x) = \int_0^x \int_0^\xi e^{-t^2}\, dt d\xi.$$

3. (Exercise 13) Find a function which is differentiable, say at every point on the interval $(-1, 1)$, but the derivative is not a continuous function.

Chapter 4 Partial Derivatives

4. (4.1.1) Show $f : \mathbb{R}^2 \to \mathbb{R}^1$ by

$$f(x, y) = \begin{cases} x^2/(x^2 + y^2), & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

   is discontinuous at $(0, 0)$ and calculate the first order partial derivatives of f on the punctured plane $\mathbb{R}^2 \backslash \{(0, 0)\}$.

5. (4.1.6) Given $u(x, y) = e^x \cos y$, find

   (a) Show that the mixed partials

$$\frac{\partial^2 u}{\partial x \partial y} \quad \text{and} \quad \frac{\partial^2 u}{\partial y \partial x}$$

   are equal. What conditions imply the equality of (second order) mixed partials in general?

   (b) Show u satisfies Laplace's equation.

   (c) Find a harmonic conjugate of u. (Hint: See page 14 of the Introduction.)

6. (5.1.1) Show

$$\int 2\sin\theta\cos\theta\,d\theta = \sin^2\theta$$
$$\int 2\sin\theta\cos\theta\,d\theta = -\cos^2\theta$$
$$\int 2\sin\theta\cos\theta\,d\theta = -\cos(2\theta)/2.$$

How can all three of these be correct?

7. (5.1.2) Show

$$\int (x^2+a^2)^{-1/2}\,dx = \sinh^{-1}(x/a)$$
$$\int (x^2+a^2)^{-1/2}\,dx = \ln(x+\sqrt{x^2+a^2}).$$