Problem 9.4.34 (Laplace's PDE) and 9.4.46 (chain rule)

```
u[x_, y_] = E^(x^2 - y^2) Cos[2 x y]
```

$$e^{x^2-y^2}\,\text{Cos}[2\,x\,y]$$

```
D[u[x, y], x]
```

$$2\,e^{x^2-y^2}\,x\,\text{Cos}[2\,x\,y] - 2\,e^{x^2-y^2}\,y\,\text{Sin}[2\,x\,y]$$

```
D[u[x, y], {x, 2}]
```

$$\left(2\,e^{x^2-y^2} + 4\,e^{x^2-y^2}\,x^2\right)\text{Cos}[2\,x\,y] - 4\,e^{x^2-y^2}\,y^2\,\text{Cos}[2\,x\,y] - 8\,e^{x^2-y^2}\,x\,y\,\text{Sin}[2\,x\,y]$$

```
D[u[x, y], y]
```

$$-2\,e^{x^2-y^2}\,y\,\text{Cos}[2\,x\,y] - 2\,e^{x^2-y^2}\,x\,\text{Sin}[2\,x\,y]$$

```
D[u[x, y], {y, 2}]
```

$$-4\,e^{x^2-y^2}\,x^2\,\text{Cos}[2\,x\,y] + \left(-2\,e^{x^2-y^2} + 4\,e^{x^2-y^2}\,y^2\right)\text{Cos}[2\,x\,y] + 8\,e^{x^2-y^2}\,x\,y\,\text{Sin}[2\,x\,y]$$

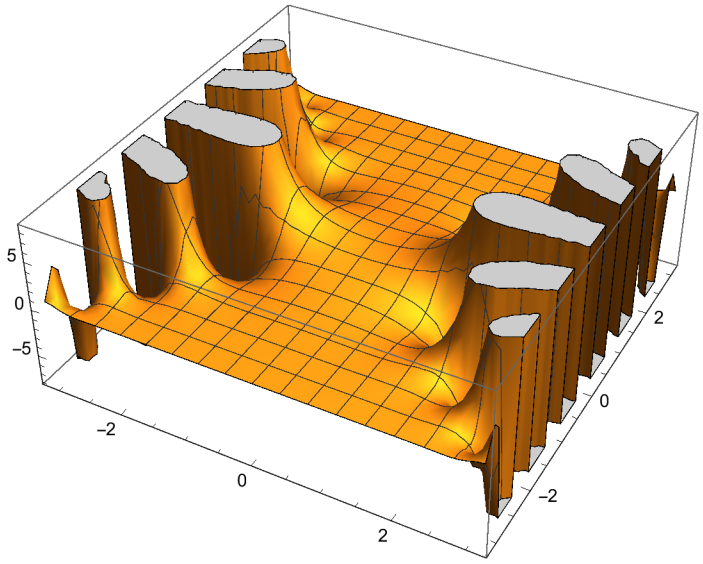
```
D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}]
```

$$-4\,e^{x^2-y^2}\,x^2\,\text{Cos}[2\,x\,y] + \left(2\,e^{x^2-y^2} + 4\,e^{x^2-y^2}\,x^2\right)\text{Cos}[2\,x\,y] - 4\,e^{x^2-y^2}\,y^2\,\text{Cos}[2\,x\,y] + \left(-2\,e^{x^2-y^2} + 4\,e^{x^2-y^2}\,y^2\right)\text{Cos}[2\,x\,y]$$

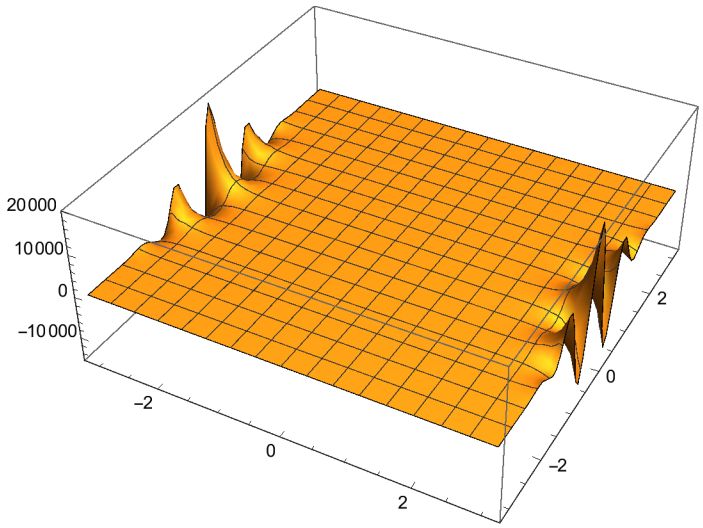
```
Simplify[D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}]]
```

$$0$$

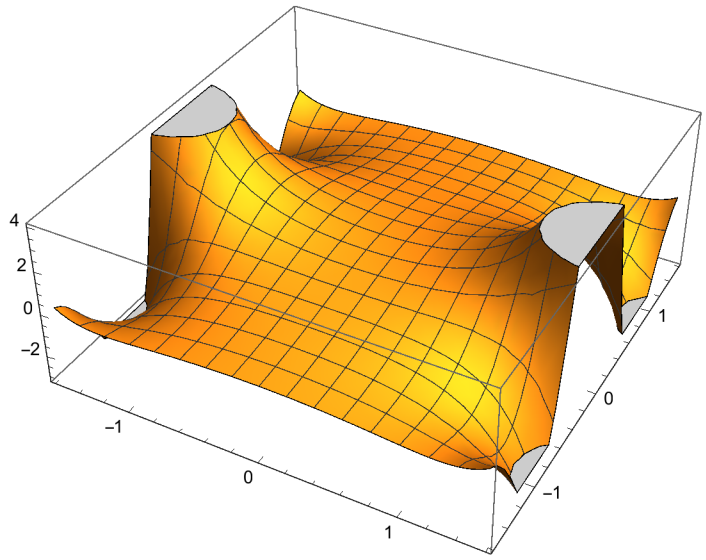
```
Plot3D[u[x, y], {x, -Pi, Pi}, {y, -Pi, Pi}]
```



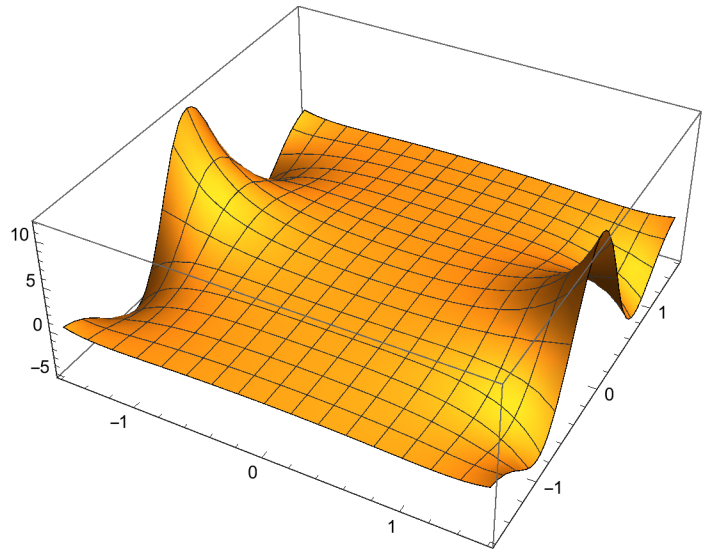

```
Plot3D[u[x, y], {x, -Pi, Pi}, {y, -Pi, Pi}, PlotRange → All]
```



```
Plot3D[u[x, y], {x, -Pi / 2, Pi / 2}, {y, -Pi / 2, Pi / 2}, AspectRatio → Automatic]
```




```
Plot3D[u[x, y], {x, -Pi / 2, Pi / 2},
 {y, -Pi / 2, Pi / 2}, AspectRatio → Automatic, PlotRange → All]
```



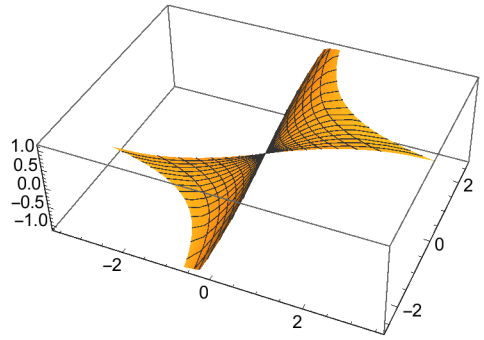

```
p[x_, t_] = t^2 ArcSin[x]
q[x_, t_] = x / t^2
r[x_, t_] = ArcTan[x / t]
```

$t^2 \text{ArcSin}[x]$

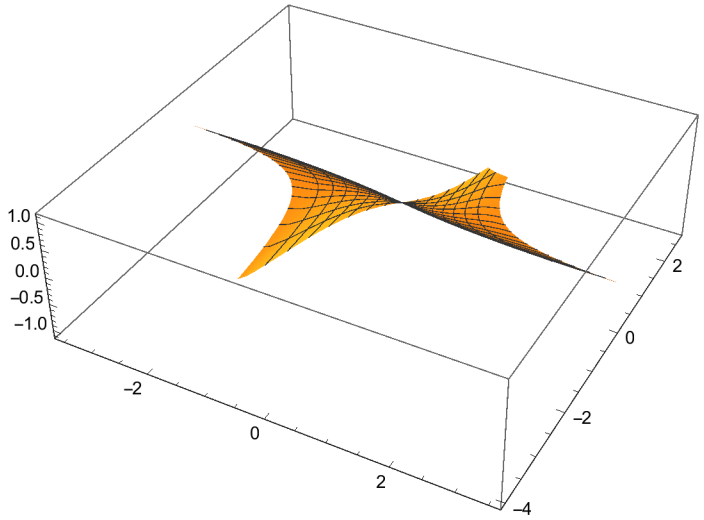
$\frac{x}{t^2}$

$\text{ArcTan}\left[\frac{x}{t}\right]$

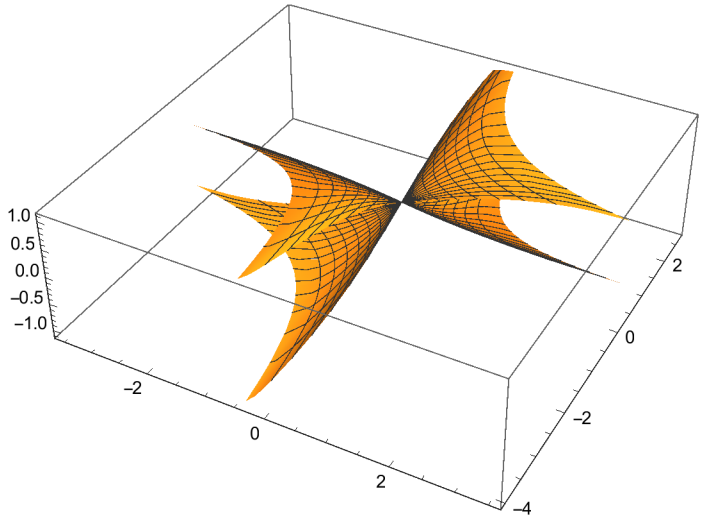
```
tophalf = ParametricPlot3D[{p[x, t], q[x, t], r[x, t]}, {x, -1, 1}, {t, 0.5, 1.5}]
```



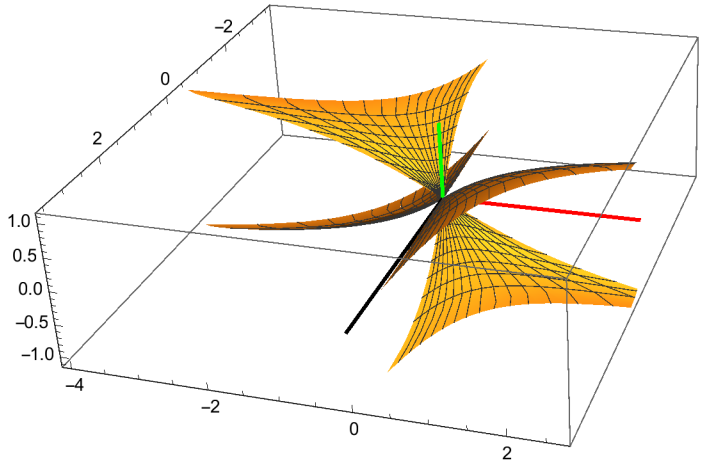

```
bottomhalf = ParametricPlot3D[{p[x, t], q[x, t], r[x, t]}, {x, -1, 1}, {t, -1.5, -0.5}]
```



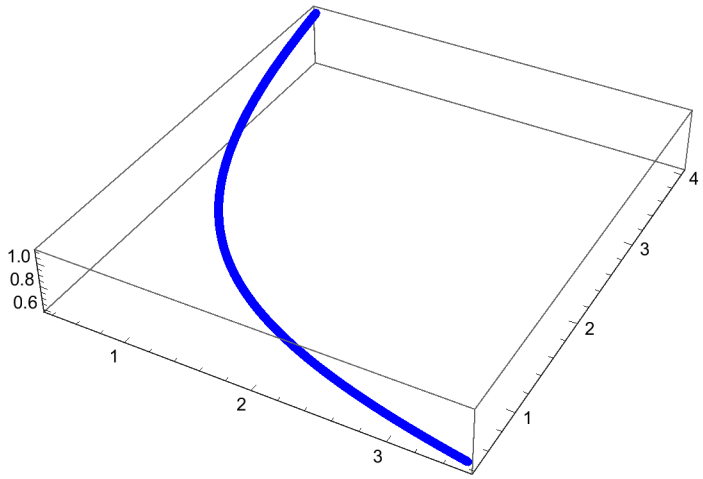

```
Show[bottomhalf, tophalf]
```



```
Show[bottomhalf, tophalf, ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}},
  {t, 0, 4}, PlotStyle → {{Thick, Black}, {Thick, Red}, {Thick, Green}}]]
```




```
curve = ParametricPlot3D[{p[1, t], q[1, t], r[1, t]},
  {t, 0.5, 1.5}, PlotStyle → {AbsoluteThickness[4.7], Blue}]
```



```
Show[bottomhalf, tophalf, ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}},
  {t, 0, 4}, PlotStyle → {{Thick, Black}, {Thick, Red}, {Thick, Green}}], curve]
```