Fourier Series Solution 13.5.14

In[1]:=
```
a = 3
b = 1
f[y_] = Sin[3 Pi y/b]
g[y_] = Sin[2 Pi y/b]
```

Out[1]= 3

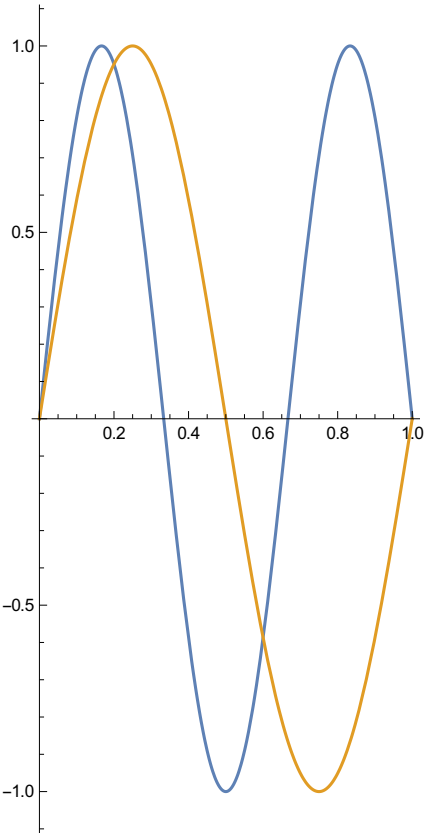
Out[2]= 1

Out[3]= Sin[3 π y]

Out[4]= Sin[2 π y]

In[5]:=
```
Plot[{f[y], g[y]}, {y, 0, b}, AspectRatio → Automatic]
```

Out[5]=



In[6]:=
```
B[j_, y_] = Sin[j Pi y/b]
```

Out[6]= Sin[j π y]

In[7]:=
```
bj[j_] = -(2/b) Coth[j Pi a/b] Integrate[ f[y] B[j, y], {y, 0, b}]
```
Out[7]=
$$-\frac{6\,\text{Coth}[3\,j\,\pi]\,\text{Sin}[j\,\pi]}{9\,\pi - j^2\,\pi}$$

In[8]:=
```
b3 = -(2/b) Coth[3 Pi a/b] Integrate[ f[y] B[3, y], {y, 0, b}]
```
Out[8]=
$$-\text{Coth}[9\,\pi]$$

In[9]:=
```
tbj[j_] = (2/b) Csch[j Pi a/b] Integrate[ g[y] B[j, y], {y, 0, b}]
```
Out[9]=
$$\frac{4\,\text{Csch}[3\,j\,\pi]\,\text{Sin}[j\,\pi]}{\left(-4 + j^2\right)\pi}$$

In[10]:=
```
tb2 = (2/b) Csch[2 Pi a/b] Integrate[ g[y] B[2, y], {y, 0, b}]
```
Out[10]=
$$\text{Csch}[6\,\pi]$$

In[11]:=
```
A[j_, x_] =
 bj[j] (- Tanh[j Pi a/b] Cosh[j Pi x/b] + Sinh[j Pi x/b]) + tbj[j] Sinh[j Pi x/b]
```
Out[11]=
$$\frac{4\,\text{Csch}[3\,j\,\pi]\,\text{Sin}[j\,\pi]\,\text{Sinh}[j\,\pi\,x]}{\left(-4 + j^2\right)\pi} - \frac{1}{9\,\pi - j^2\,\pi}$$
$$6\,\text{Coth}[3\,j\,\pi]\,\text{Sin}[j\,\pi]\left(\text{Sinh}[j\,\pi\,x] - \text{Cosh}[j\,\pi\,x]\,\text{Tanh}[3\,j\,\pi]\right)$$

In[12]:=
```
A3[x_] = b3 (- Tanh[3 Pi a/b] Cosh[3 Pi x/b] + Sinh[3 Pi x/b]) + tbj[3] Sinh[3 Pi x/b]
```
Out[12]=
$$-\text{Coth}[9\,\pi]\left(\text{Sinh}[3\,\pi\,x] - \text{Cosh}[3\,\pi\,x]\,\text{Tanh}[9\,\pi]\right)$$

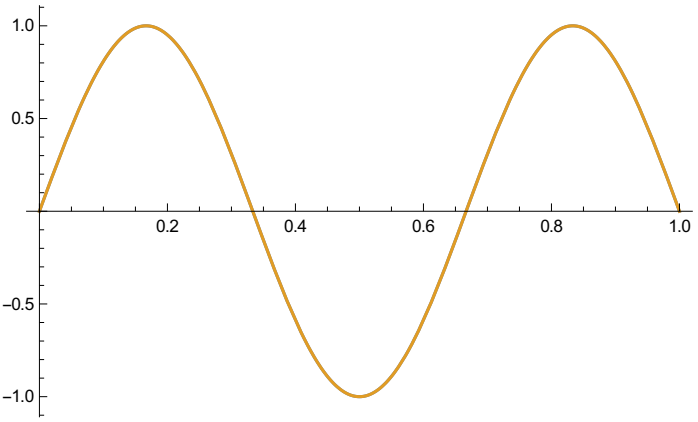
In[13]:=
```
A2[x_] = bj[2] (- Tanh[2 Pi a/b] Cosh[2 Pi x/b] + Sinh[2 Pi x/b]) + tb2 Sinh[2 Pi x/b]
```
Out[13]=
$$\text{Csch}[6\,\pi]\,\text{Sinh}[2\,\pi\,x]$$

In[14]:=
```
u[x_, y_, k_] := Sum[A[j, x] B[j, y], {j, 1, k}]
```

In[15]:=
```
u23[x_, y_] = A2[x] B[2, y] + A3[x] B[3, y]
```
Out[15]=
$$\text{Csch}[6\,\pi]\,\text{Sin}[2\,\pi\,y]\,\text{Sinh}[2\,\pi\,x] - \text{Coth}[9\,\pi]\,\text{Sin}[3\,\pi\,y]\left(\text{Sinh}[3\,\pi\,x] - \text{Cosh}[3\,\pi\,x]\,\text{Tanh}[9\,\pi]\right)$$

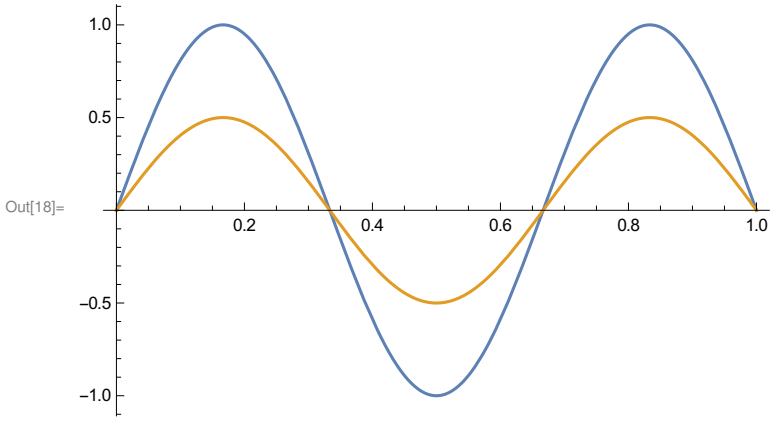
In[16]:=

In[17]:=
```
Plot[{f[y], u23[0, y]}, {y, 0, b}]
```
Out[17]=



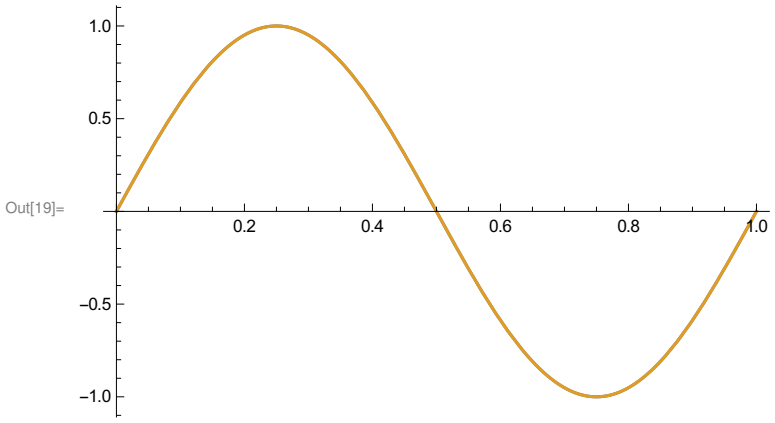
In[18]:=
```
Plot[{f[y], u23[0, y] / 2}, {y, 0, b}]
```

Out[18]=



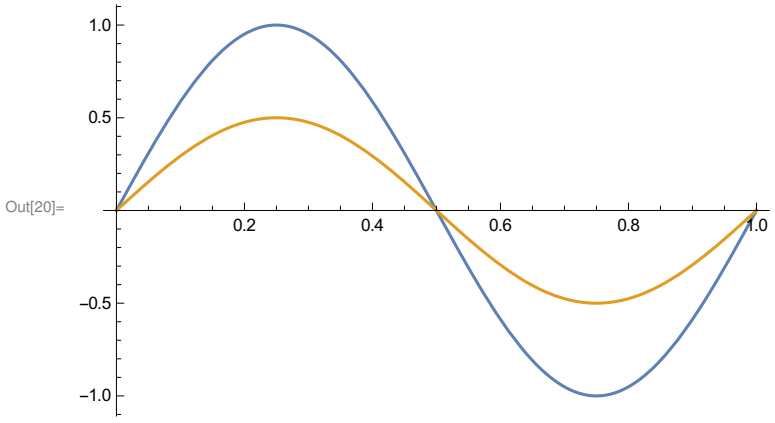
In[19]:=
```
Plot[{g[y], u23[a, y]}, {y, 0, b}]
```

Out[19]=



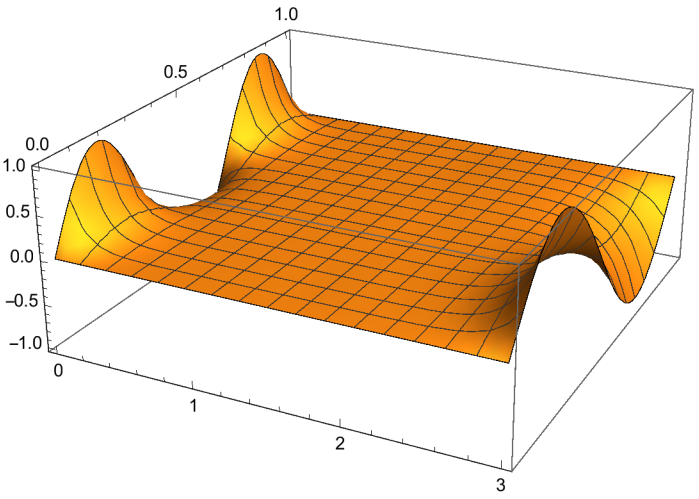
In[20]:=
```
Plot[{g[y], u23[a, y] / 2}, {y, 0, b}]
```

Out[20]=



In[21]:=
```
Plot3D[u23[x, y], {x, 0, a}, {y, 0, b}, PlotRange → All]
```

Out[21]=



In[22]:=
```
Animate[Plot[u23[x, y], {y, 0, b}, PlotRange → {-1, 1}], {x, 0, a}]
```

Out[22]=