

Fourier Series Solution 13.5.11

```
In[33]:= a = 3  
f[x_] = x (x - a)
```

```
Out[33]= 3
```

```
Out[34]= (-3 + x) x
```

```
In[35]:= A[j_, x_] = Sin[j Pi x / a]
```

```
Out[35]= Sin[ $\frac{j \pi x}{3}$ ]
```

```
In[36]:= bj[j_] = (2 / a) Integrate[f[x] A[j, x], {x, 0, a}]
```

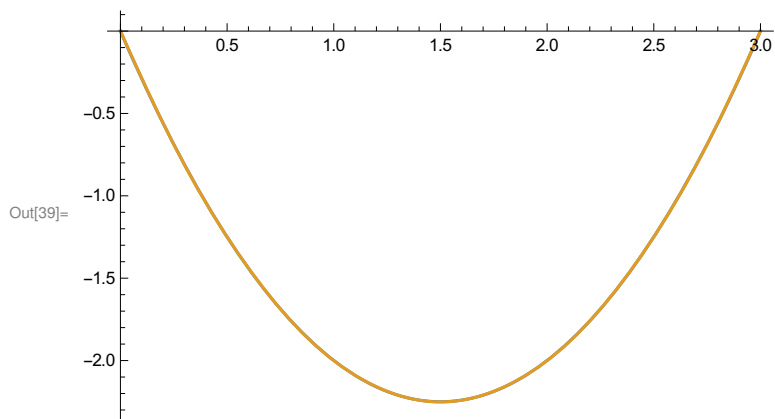
```
Out[36]=  $\frac{18 (-2 + 2 \cos[j \pi] + j \pi \sin[j \pi])}{j^3 \pi^3}$ 
```

```
In[37]:= B[j_, y_] = bj[j] E^(-j Pi y / a)
```

```
Out[37]=  $\frac{18 e^{-\frac{1}{3} j \pi y} (-2 + 2 \cos[j \pi] + j \pi \sin[j \pi])}{j^3 \pi^3}$ 
```

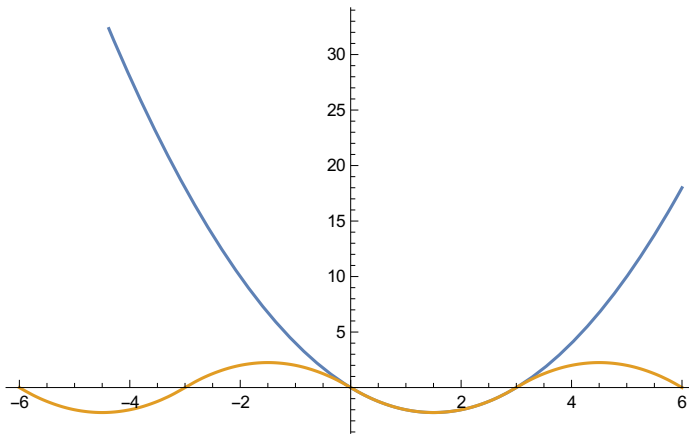
```
In[38]:= u[x_, y_, k_] := Sum[A[j, x] B[j, y], {j, 1, k}]
```

```
In[39]:= Plot[{f[x], u[x, 0, 10]}, {x, 0, a}]
```



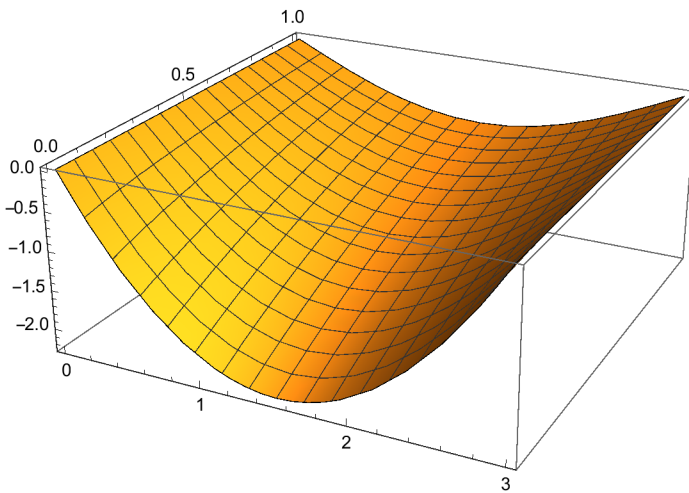
```
In[40]:= Plot[{f[x], u[x, 0, 10]}, {x, -2 a, 2 a}]
```

Out[40]=

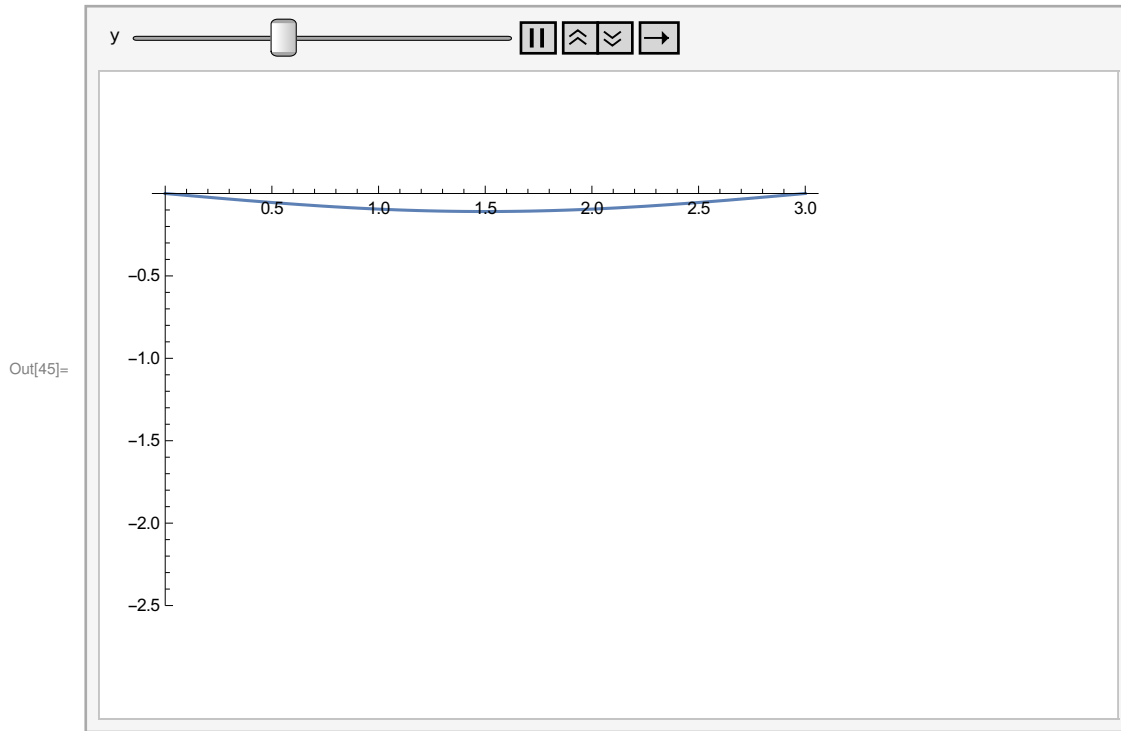


```
In[41]:= Plot3D[u[x, y, 10], {x, 0, a}, {y, 0, 1}, PlotRange -> All]
```

Out[41]=



```
In[45]:= Animate[Plot[u[x, y, 10], {x, 0, a}, PlotRange -> {-2.5, 0}], {y, 0, 3}]
```



Compare to heat equation

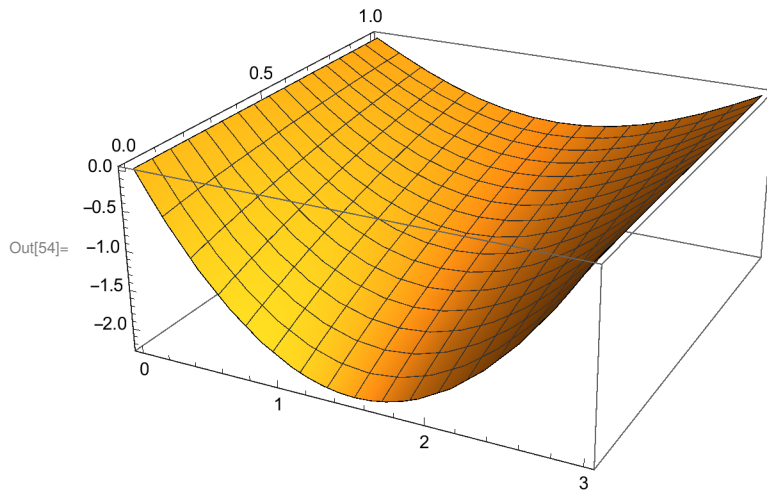
```
In[46]:= HB[j_, t_] = bj[j] E^(-Sqrt[j Pi/a] t)
```

```
Out[46]= 
$$\frac{18 e^{-\sqrt{j} \sqrt{\frac{\pi}{3}} t} (-2 + 2 \cos[j \pi] + j \pi \sin[j \pi])}{j^3 \pi^3}$$

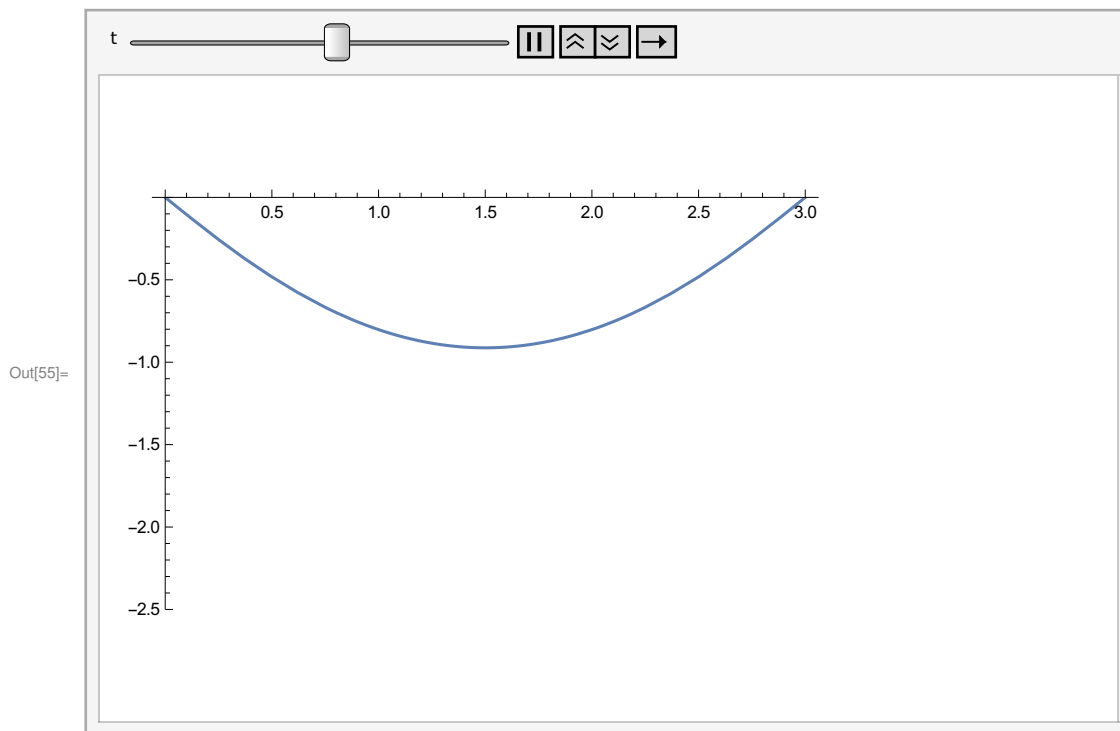
```

```
In[53]:= hu[x_, t_, k_] := Sum[A[j, x] HB[j, t], {j, 1, k}]
```

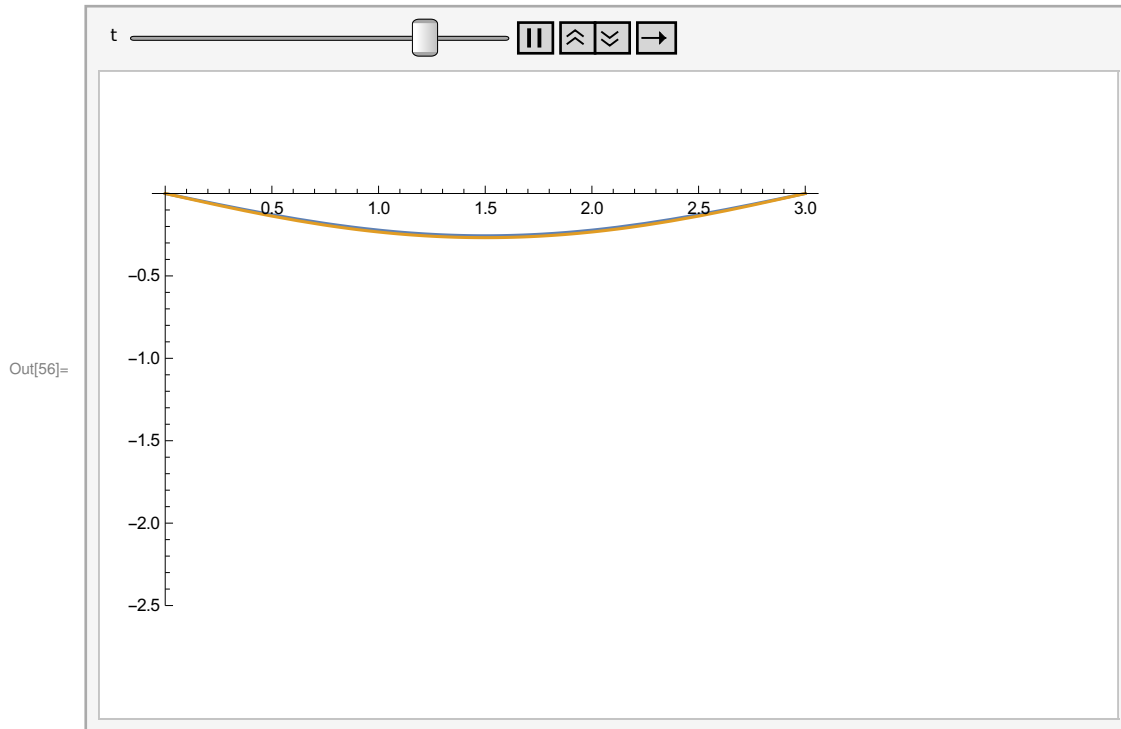
```
In[54]:= Plot3D[hu[x, t, 10], {x, 0, a}, {t, 0, 1}, PlotRange -> All]
```



```
In[55]:= Animate[Plot[hu[x, t, 10], {x, 0, a}, PlotRange -> {-2.5, 0}], {t, 0, 3}]
```



```
In[56]:= Animate[Plot[{u[x, t, 10], hu[x, t, 10]}, {x, 0, a}, PlotRange → {-2.5, 0}], {t, 0, 3}]
```



```
In[58]:= Animate[
  Plot[{u[x, t, 10], hu[x, t, 10]}, {x, 0, a/10}, PlotRange → {-0.5, 0}], {t, 0, 3}]
```

