

Fourier Series Solution 13.3.5

Coefficients to represent the even extension

```
In[15]:= f[x_] = x^3 + 1
```

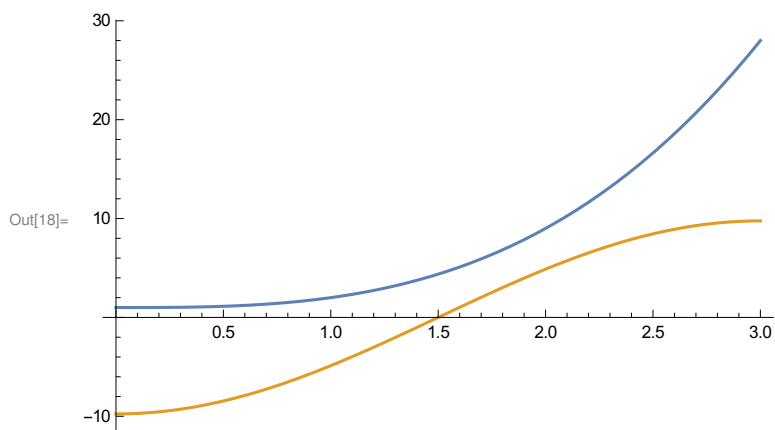
```
Out[15]= 1 + x^3
```

```
In[16]:= a[j_, L_] = (2 / L) Integrate[f[x] Cos[j Pi x / L], {x, 0, L}]
```

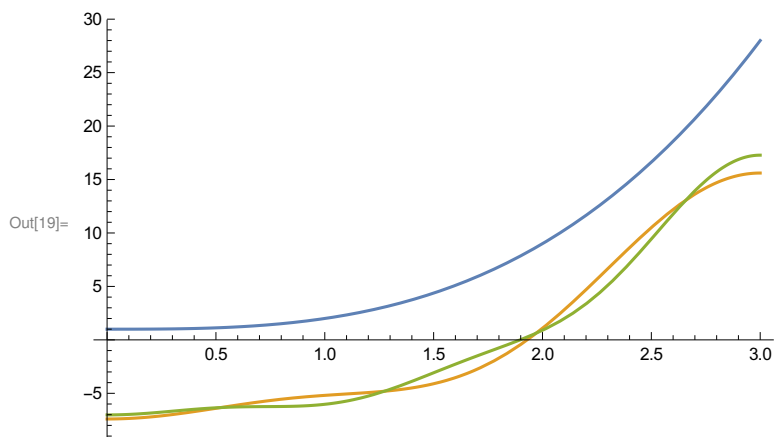
```
Out[16]=  $\frac{1}{j^4 \pi^4} 2 (j^3 \pi^3 \sin[j \pi] + L^3 (6 + 3 (-2 + j^2 \pi^2) \cos[j \pi] + j \pi (-6 + j^2 \pi^2) \sin[j \pi]))$ 
```

```
In[17]:= fk[x_, k_, L_] := Sum[a[j, L] Cos[j Pi x / L], {j, 1, k}]
```

```
In[18]:= Plot[{f[x], fk[x, 1, 3]}, {x, 0, 3}]
```



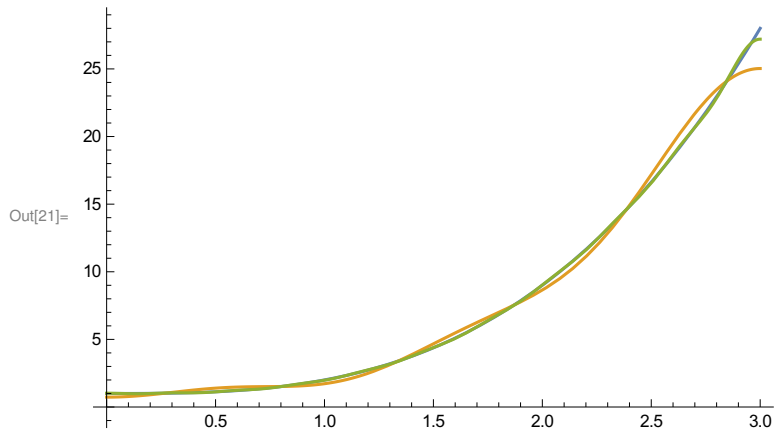
```
In[19]:= Plot[{f[x], fk[x, 3, 3], fk[x, 5, 3]}, {x, 0, 3}]
```



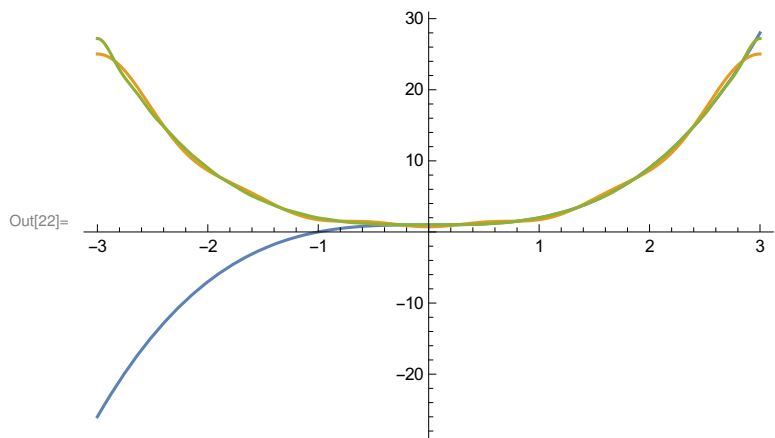
In[20]:= `azero[L_] = (1 / L) Integrate[f[x], {x, 0, L}]`

Out[20]=
$$\frac{L + \frac{L^4}{4}}{L}$$

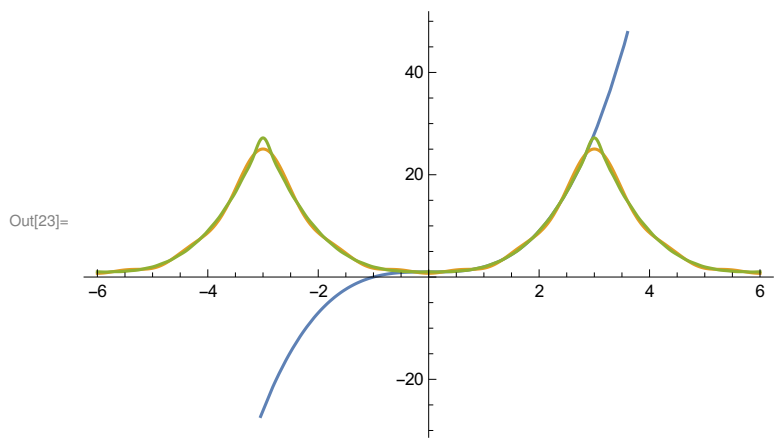
In[21]:= `Plot[{f[x], fk[x, 5, 3] + azero[3], fk[x, 20, 3] + azero[3]}, {x, 0, 3}]`



In[22]:= `Plot[{f[x], fk[x, 5, 3] + azero[3], fk[x, 20, 3] + azero[3]}, {x, -3, 3}]`



In[23]:= `Plot[{f[x], fk[x, 5, 3] + azero[3], fk[x, 20, 3] + azero[3]}, {x, -6, 6}]`



```
In[40]:= B[t_, j_, L_] = E^((-j^2 Pi^2 / L^2 - 1) t)
```

```
Out[40]= e(-1 -  $\frac{j^2 \pi^2}{L^2}$ ) t
```

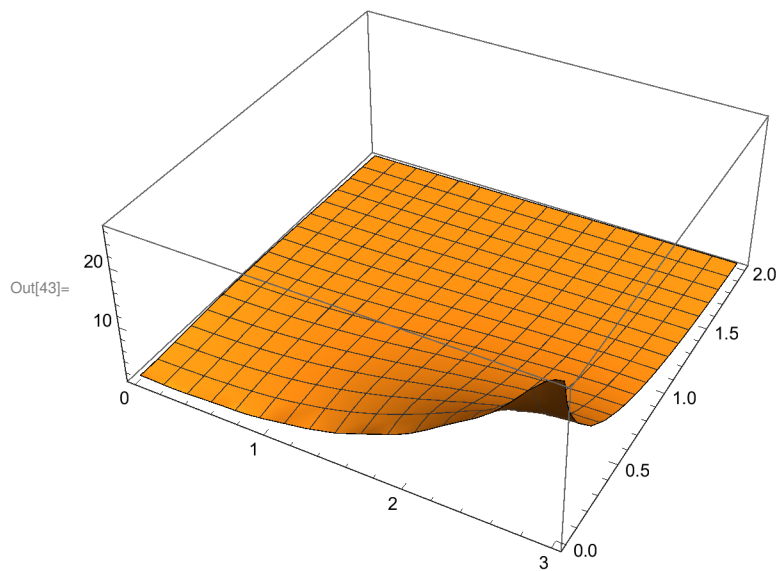
```
In[25]:= Bzero[t_] = E^(-t)
```

```
Out[25]= e-t
```

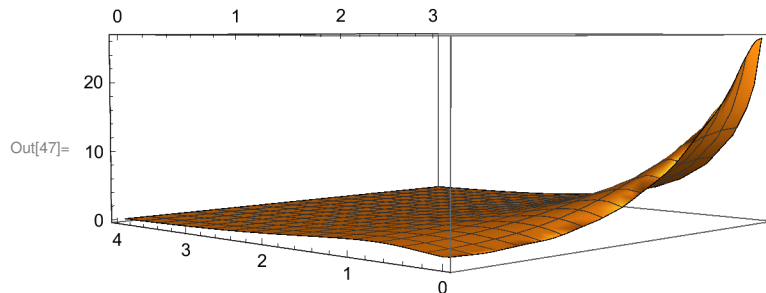
solution

```
In[41]:= u[x_, t_, k_, L_] := Sum[a[j, L] B[t, j, L] Cos[j Pi x / L], {j, 1, k}] + azero[3] Bzero[t]
```

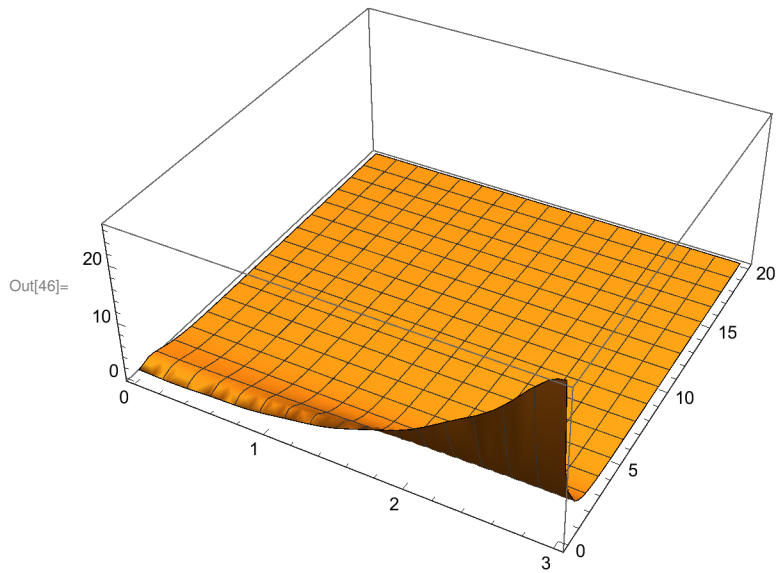
```
In[43]:= Plot3D[u[x, t, 10, 3], {x, 0, 3}, {t, 0, 2}, PlotRange -> All]
```



```
In[47]:= Plot3D[u[x, t, 10, 3], {x, 0, 3}, {t, 0, 4}, PlotRange -> All]
```



```
In[46]:= Plot3D[u[x, t, 10, 3], {x, 0, 3}, {t, 0, 20}, PlotRange -> All]
```



```
In[49]:= Plot[{1 - t, u[0, t, 100, 3]}, {t, 0, 0.1}]
```

