Fourier Series Solution 13.2.11

Coefficients to represent the even extension

```
f[y_] = 1
```
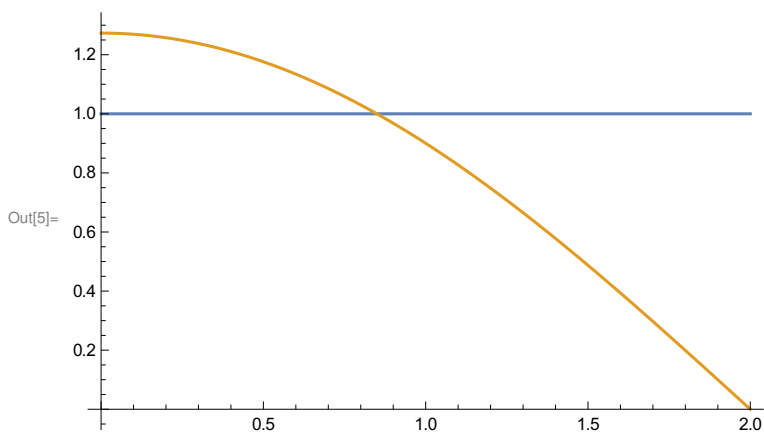
Out[1]= 1

In[2]:= `a[j_, L_] = (2 / L) Integrate[f[y] Cos[(1 + 2 j) Pi y / (2 L)], {y, 0, L}]`

Out[2]= $\dfrac{4 \, Cos[j \, \pi]}{\pi + 2 \, j \, \pi}$

In[3]:= `fk[y_, k_, L_] := Sum[a[j, L] Cos[(2 j + 1) Pi y / (2 L)], {j, 0, k}]`

In[5]:= `Plot[{f[y], fk[y, 0, 2]}, {y, 0, 2}]`

Out[5]=

In[7]:= `Plot[{f[y], fk[y, 1, 2], fk[y, 2, 2]}, {y, 0, 2}, AxesOrigin → {0, 0}]`

Out[7]=
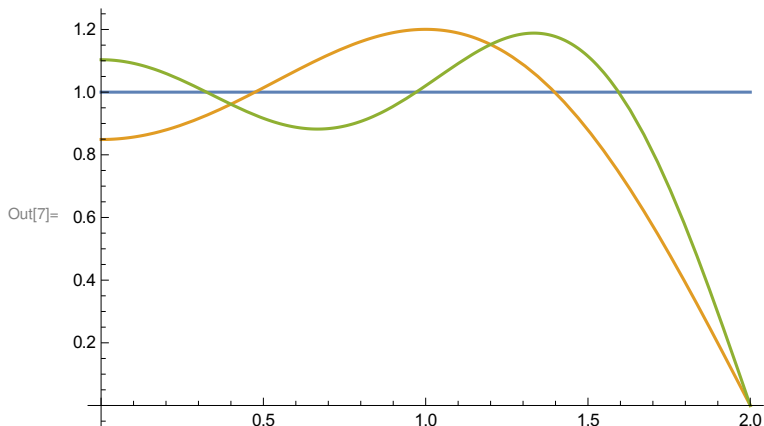
In[9]:= `Plot[{f[y], fk[y, 5, 2], fk[y, 20, 2]}, {y, 0, 2}, AxesOrigin → {0, 0}, PlotRange → All]`

Out[9]=
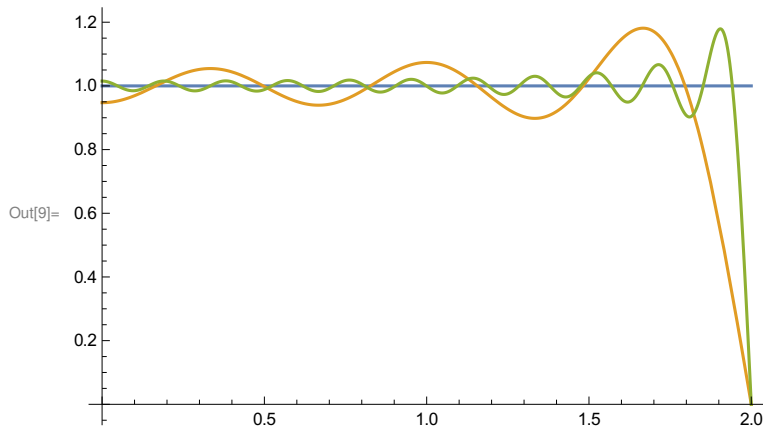
In[11]:= `Plot[{f[y], fk[y, 5, 2], fk[y, 20, 2]}, {y, -8, 8}, AxesOrigin → {0, 0}, PlotRange → All]`

Out[11]=
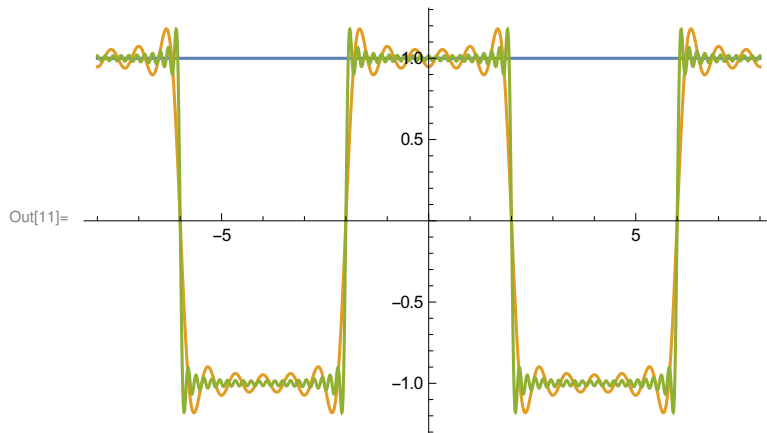
In[18]:= `b[j_, L_, M_] = a[j, L] / Cosh[(2 j + 1) Pi M / (2 L)]`

Out[18]= $\dfrac{4 \, \text{Cos}[j \, \pi] \, \text{Sech}\left[\frac{(1+2 \, j) \, M \, \pi}{2 \, L}\right]}{\pi + 2 \, j \, \pi}$

In[13]:= `B[y_, j_, L_] = Cos[(2 j + 1) Pi y / (2 L)]`

Out[13]= $\text{Cos}\left[\dfrac{(1 + 2 \, j) \, \pi \, y}{2 \, L}\right]$

In[14]:= `A[x_, j_, L_] = Cosh[(2 j + 1) Pi x / (2 L)]`

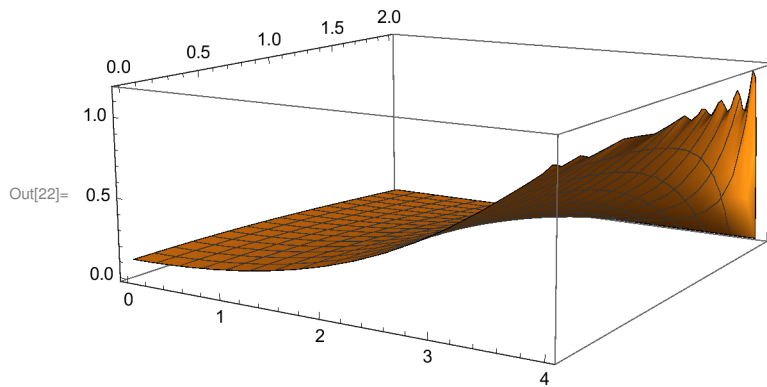Out[14]= $\text{Cosh}\left[\dfrac{(1 + 2 \, j) \, \pi \, x}{2 \, L}\right]$

solution

In[20]:= `u[x_, y_, k_, L_, M_] := Sum[b[j, L, M] B[y, j, L] A[x, j, L], {j, 0, k}]`

In[21]:= `Plot3D[u[x, y, 10, 2, 4], {x, 0, 4}, {y, 0, 2}, PlotRange → All]`

Out[21]=
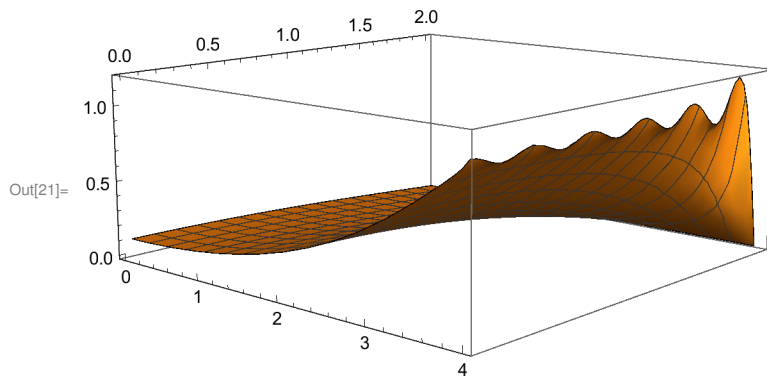
In[22]:= `Plot3D[u[x, y, 20, 2, 4], {x, 0, 4}, {y, 0, 2}, PlotRange → All]`

Out[22]=
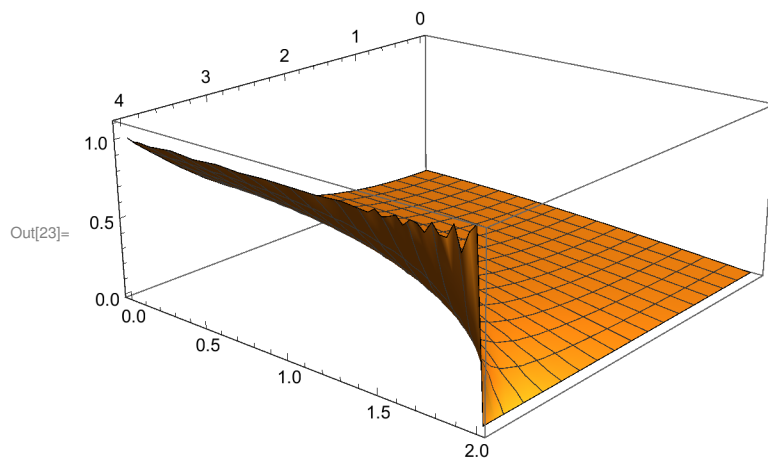
In[23]:= `Plot3D[u[x, y, 40, 2, 4], {x, 0, 4}, {y, 0, 2}, PlotRange → All]`

Out[23]=

In[24]:= `Plot3D[u[x, y, 60, 2, 2], {x, 0, 2}, {y, 0, 2}, PlotRange → All]`

Out[24]=