

Pythagoras	c.500 BC	
Euclid	c.300 BC	Euclidean geometry
Archimedes	c.287–212 BC	
Eratosthenes	c.276–195 BC	
Diophantus	c. 210–290 AD	
René Descartes	1596–1650	
Pierre de Fermat	1607–1665	Cartesian geometry
Isaac Newton	1642–1726	Newtonian geometry
		Cayleyan geometry
Carl Gauss	1777–1855 (1828)	
Bernhard Riemann	1826–1866 (1854)	Gaussian geometry
Karl Weierstrass	1815–1897	
Arthur Cayley	1821–1895	
Henri Poincaré	1854–1912	
David Hilbert	1862–1943	Riemannian geometry
Hermann Weyl	1885–1955 (1913)	
Hassler Whitney	1907–1989 (1936)	
Élie Cartan	1869–1951	
Albert Einstein	1879–1955	

Table 1.1: Some mathematicians with ideas relevant to geometry and some kinds of geometry

Indeed many have pointed out the large gaps in time between the introduction of the basic ideas of Riemannian geometry by Riemann in 1854, the appearance of an axiomatic formulation of those ideas by Hermann Weyl in 1913, and the rise in interest in the subject starting with the results of Has-

sler Whitney starting in 1936. The ideas underlying Riemannian geometry are difficult, and it is far from clear (to me) how to present and/or motivate them. What seems to have been adopted as something of a standard presentation starts with a statement of the definition of a Riemannian manifold without much critical consideration<sup>1</sup> followed by a retreat into pushing indices up and down and around. I would like to try something different.

## 1.2 Euclid: point, line, and plane

A **point** is that which has no part.

—Euclid (c.300 BC)

Historically, as a victim of post-classical western culture, I have not been impressed with Euclid’s definition of a point. It is not a definition given in terms of either previously defined terms nor some manner of axiomatic linguistic formulations. One can ask (derisively or insincerely): What is this actually saying?

In retrospect, however, I think this definition has some quite interesting and maybe even relevant aspects. This definition might even be considered somewhat profound. As a start, one can ask the same question sincerely:

What is this saying; what is the intent?

Taking Spengler’s advice that Euclid had an entirely different mind or conception of reality based in the positive presence of a “body” rather than the abstract “absence” of space natural to my mind through indoctrination, one view of the intent might be “indivisibility.” Euclid wishes to consider something, some “body” say, geometrically which cannot be broken into parts. On the one hand, the profound aspects of this idea of an “indivisible body” strike me as lying in spheres of thought rather distinct from geometry. The simplest sphere would be the physical sphere of the human body, with respect to which one may say: A human body is “whole” meaning metaphorically “healthy.” Again, simplistically, no parts have been cut off or divided. Were parts to have been cut off, then the body itself as a healthy whole may be considered to have ceased to exist. This physical metaphor linking the human body to “that which has no part” is relatively old:

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<sup>1</sup>... or even worse a topological manifold