Math 6342, Final Exam (practice) Name and section: ____________________

The problems on this exam are related to the notes on the "main existence and uniqueness theorem" for linear elliptic PDE. These notes are posted at

`http://www.math.gatech.edu/ mccuan/courses/6342/existence.pdf`.

1. (33+1/3 points) (weak supersolutions)

   A Define what is meant by a weak supersolution for the operator

$$Lu = -\sum_{i,j} D_i(a_{ij}D_j u) + \sum_j b_j D_j u + cu.$$

   B Show that the expression $\inf\{M : (u^+ - M)^+ \in H_0^1(\mathcal{U})\}$ appearing in the weak maximum principle in the notes is the same as

$$\inf\{M : \max\{u, M\} - M \in H_0^1(\mathcal{U})\}.$$

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2. (33+1/3 points) (weak supersolutions)

A Formulate a *weak minimum principle for weak supersolutions.* Be sure to include all necessary hypotheses and define all conditions which require special attention, e.g., what is the infemum of u on the boundary of a domain?

B Give a detailed proof of the uniqueness of weak solutions asserted in Corollary 1 of the notes.

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3. (33+1/3 points) In the proof of the main existence theorem, we used the uniqueness which follows from the weak maximum/minimum principle to assert that the solution we obtained was unique. This required $c \geq 0$.

   For this problem, I want you to set aside the condition $c \geq 0$. As a consequence the first part of the proof of the main theorem is no longer valid. Nevertheless, we can still use the Fredholm theorem, and the argument showing existence of a solution when the second alternative holds is still perfectly valid.

   Assuming we are in the second case of the Fredholm alternative, give an alternative proof of uniqueness of the solution shown to exist in the notes. (Hint: Use the uniqueness asserted in the Fredhold alternative itself.)