1. Use the dominated convergence theorem for real valued functions to prove the dominated convergence theorem for complex valued functions.

2. Let $\mathscr{M}_\alpha = \mathcal{M}(\mathcal{E}_\alpha)$ be the σ-algebra on X_α generated by the sets in $\mathcal{E}_\alpha$ where $\alpha \in \Gamma$ and Γ is a countable indexing set. Define
$$\mathcal{E}_0 = \cup_{\alpha\in\Gamma}\{\pi_\alpha^{-1}E : E \in \mathcal{E}_\alpha\}.$$
Show that
$$\tilde{\mathscr{M}} = \{A \subset X_\alpha : \pi_\alpha^{-1}A \in \mathcal{M}(\mathcal{E}_0)\}$$
is a σ-algebra.

3. Every separable metric space is second countable; $\mathbb{R}^n$ is second countable.

4. Given two measure spaces $X_1, \mathscr{M}_1$ and $X_2, \mathscr{M}_2$, define
$$\tilde{\mathscr{M}} = \{\tilde{E} \subset X_1 \times X_2 : \{x \in X_1 : (x, x_2) \in \tilde{E}\} \in \mathscr{M}_1 \text{ for all } x_2 \in X_2\}.$$
Show that $\tilde{\mathscr{M}}$ is a σ-algebra containing all rectangles.