1. If $\mathcal{B} \subset \mathcal{P}(X)$ is closed under complements and finite unions (i.e., is an algebra) and is also closed under countable disjoint unions, then $\mathcal{B}$ is closed under countable unions (i.e., is a σ-algebra).

2. Assume that
$$\mu_* A = \max_{A \supset E \in \mathcal{A}} \mu E$$
where μ is a measure on $\mathcal{A}$, i.e., given $A \subset X$, find a set $E_* \in \mathcal{A}$ such that $E_* \subset A$ and $\mu E_* = \mu_* A$. This is inner approximation by measurable sets. Use this to simplify the proof given in class of Lemma C: The Carathéodory condition is equivalent to $\mu_* A = \mu^* A$.

3. Consider
$$\mathcal{A}_0 = \{\cup_{j=1}^{k} (a_j, b_j] : 0 \le a_1 \le b_1 \le a_2 \le b_2 \le \cdots \le a_k \le b_k \le 1\}$$
and $\mu_0 : \mathcal{A}_0 \to [0, \infty)$ by
$$\mu_0 \left(\cup_{j=1}^{k} (a_j, b_j] \right) = \sum_{j=1}^{k} (b_j - a_j).$$
   (a) Show that $\mathcal{A}_0$ is an algebra.
   (b) Show that μ_0 is well defined.
   (c) Show that μ_0 is a premeasure.
   (d) Let μ^* be the outer measure derived from the premeasure μ_0 and show that the (outer) measure of any interval is its length.