1. (n.b., 1.4.18,22) Given a measure μ defined on a sigma algebra $\mathcal{A}$ of subsets of X, define $\mu^* : \mathcal{P}(X) \to [0, \infty]$ by

$$\mu^* E = \inf_{\mathcal{A} \ni A \supset E} \mu A.$$

Show the following:

(a) μ^* is an outer measure.

(b) $\mu^*(Z) = 0$ if and only if there exists $A \in \mathcal{A}$ with $Z \subset A$ and $\mu A = 0$.

(c) Let $\bar{\mathcal{A}} = \{A \cup Z : A \in \mathcal{A} \text{ and } \mu^* Z = 0\}$, and define $\bar{\mu} : \bar{\mathcal{A}} \to [0, \infty]$ by

$$\bar{\mu}(A \cup Z) = \mu A.$$

Show that $\bar{\mathcal{A}}$ is a σ-algebra and $\bar{\mu}$ is a measure.