Assignment 1: Complex Numbers
Due Tuesday January 25, 2022

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January 9, 2022

Problem 1 *(Ahlfors 1.1.2) Let* $z = x + iy$ *with* $x, y \in \mathbb{R}$*. Find the real and imaginary parts of the following complex numbers:*

(a) z^4.

(b) $1/z$.

(c) $(z-1)/(z=1)$.

(a) $1/z^2$.

Problem 2 *(S&S Exercise 1.1) Let* z_1 *and* z_2 *be fixed complex numbers. Describe geometrically the set*

$$\{z \in \mathbb{C} : |z - z_1| = |z - z_2|\}.$$

Problem 3 *(S&S Exercise 1.2) If* $P = (x_1, y_1) \in \mathbb{R}^2$ *and* $Q = (x_2, y_2) \in \mathbb{R}^2$*, then define the dot product of* P *and* Q *by*

$$P \cdot Q = x_1 x_2 + y_1 y_2.$$

If $z, w \in \mathbb{C}$*, define the* **Hermitian product** *of* z *and* w *by*

$$(z, w) = z\overline{w}$$

where $\overline{w}$ *is the* **complex conjugate** *of* w*, that is* $\overline{w} = x_2 - iy_2$ *if* $w = x_2 + iy_2$*. Show that if* $z = x_1 + iy_1$ *and* $w = x_2 + iy_2$*, then*

$$P \cdot Q = \frac{1}{2}[(z, w) + (w, z)] = \mathrm{Re}(z, w)$$

where $P = (x_1, y_1)$ *and* $Q = (x_2, y_2)$ *are the points in* $\mathbb{R}^2$ *identified with* z *and* w *respectively.*

Problem 4 *(S&S Exercise 1.3) Solve the equation $z^n = se^{i\phi}$ where $n \in \mathbb{N}$ is a natural number, $s > 0$, and $\phi \in \mathbb{R}$.*

Note: When Stein introduces Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

on page 4, he is simply defining notation or alternatively he is defining the value of the exponential function $f(z) = e^z$ strictly along the imaginary axis. We do not know at this point anything about the value of e^z unless $z \in \mathbb{R}$ is real or $z \in i\mathbb{R}$ is purely imaginary.

Problem 5 *(Ahlfors 1.4.4) Let a, b, c be fixed complex numbers. Find conditions under which the equation*

$$az + b\overline{z} + c = 0$$

for $z \in \mathbb{C}$ has exactly one solutuion and find a formula for that solution. Hint: Cramer's Rule.