$$"-1" = \sum_{j=1}^{\infty} c_j \sin \frac{j\pi}{L} x$$

$$-\underbrace{\int_0^L \sin \frac{j\pi}{L} x \, dx}_{\frac{L}{j\pi} \cos \frac{j\pi}{L} x \Big|_0^L} = c_j \underbrace{\int_0^L \sin^2 \frac{j\pi}{L} x \, dx}_{L/2}$$

$$= -\frac{L}{j\pi}\left(1 - (-1)^j\right)$$

$$c_j = \begin{cases} -\frac{4}{j\pi} & , j = 2k+1 \text{ odd} \\ 0 & , j \text{ even.} \end{cases}$$

$$-1 = -\frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{2k+1} \sin\left(\frac{(2k+1)\pi}{L} x\right)$$

[redacted]

$$\underbrace{W_t = W_{xx} - \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{1}{2k+1}\sin\left(\frac{(2k+1)\pi}{L}x\right)}$$

↑ singular at $x=0,L$

$$\underline{W_k = B_k(t)\sin\left(\frac{(2k+1)\pi}{L}x\right)}$$

<u>Consider</u>

$$B_k'\cancel{\sin\left(\frac{(2k+1)\pi}{L}x\right)} = B_k\left[-\frac{(2k+1)^2\pi^2}{L^2}\cancel{\sin\left(\frac{(2k+1)\pi}{L}x\right)}\right]$$

$$-\frac{4}{\pi(2k+1)}\cancel{\sin\left(\frac{(2k+1)\pi}{L}x\right)}$$

[redacted]

(1) Solve for B_k

(2) $W = \sum W_k$, $u = W + t$