MATH 4581 Lecture 6 Thursday Sept. 9, 2021

<u>Last time</u>:

$$\begin{cases} u_t = k\, u_{xx} \quad \text{on } (0,L)\times(0,T) \\ u_x(0,t) = 0 = u_x(L,t), \quad 0<t<T \\ u(x,0) = g(x) \end{cases}$$

$$u(x,t) = u_0 + \sum_{j=1}^{\infty} a_j\, e^{-k\frac{j^2\pi^2}{L^2}t} \cos\frac{j\pi}{L}x$$

const.

$$u_0 = \frac{1}{L}\int_0^L g(x)\,dx, \quad a_j = \frac{2}{L}\int_0^L g(x)\cos\frac{j\pi}{L}x\,dx$$ [redacted]

Transport / Convection

transporting mass:

Start with a vector (velocity) field:

$$\mathbb{V} \quad , \quad [\mathbb{V}] = \frac{L}{T}$$

volumetric mass density ρ , $[\rho] = \frac{M}{L^3}$

$$[\rho \mathbb{V}] = \frac{M}{T L^2}$$

↖ These are dimensions of a flux

[redacted]

rate at which mass is <u>transported</u> across a surface S is

$$\int_S \rho \mathbf{v} \cdot n$$

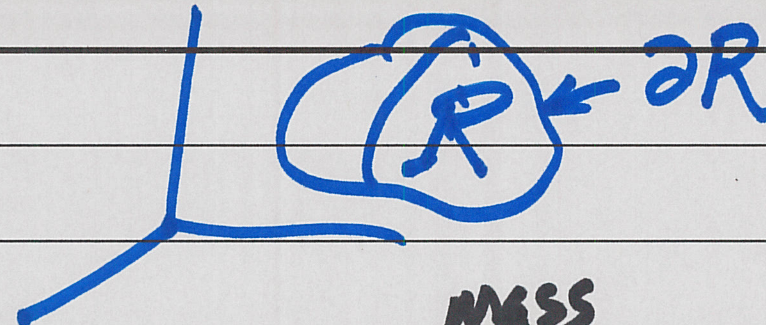
In <u>particular</u>,

$$\int_{\partial R} \rho \mathbf{v} \cdot n$$

rate of mass going <u>out</u> of R

outward <u>unit</u> normal

[redacted]



Mass Conservation:

$\rho = \rho(x, t)$ spatially dependent mass density

$$\frac{d}{dt}\int_R \rho = -\int_{\partial R} \rho \mathbb{V}\cdot\vec{n} = -\int_R \operatorname{div}(\rho \mathbb{V})$$

rate at which mass enters R

$$\boxed{\rho_t = -\operatorname{div}(\rho \mathbb{V})}$$

Continuity Equation.

[redacted]