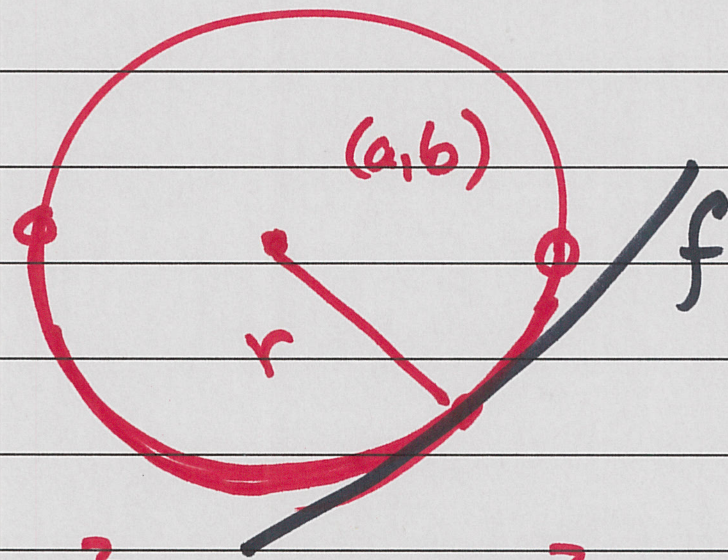



$$(x-a)^2 + (y-b)^2 = r^2$$

$$\boxed{y = b - \sqrt{r^2 - (x-a)^2}} = g(x)$$

(iii) $g''(x_0) = f''(x_0)$.

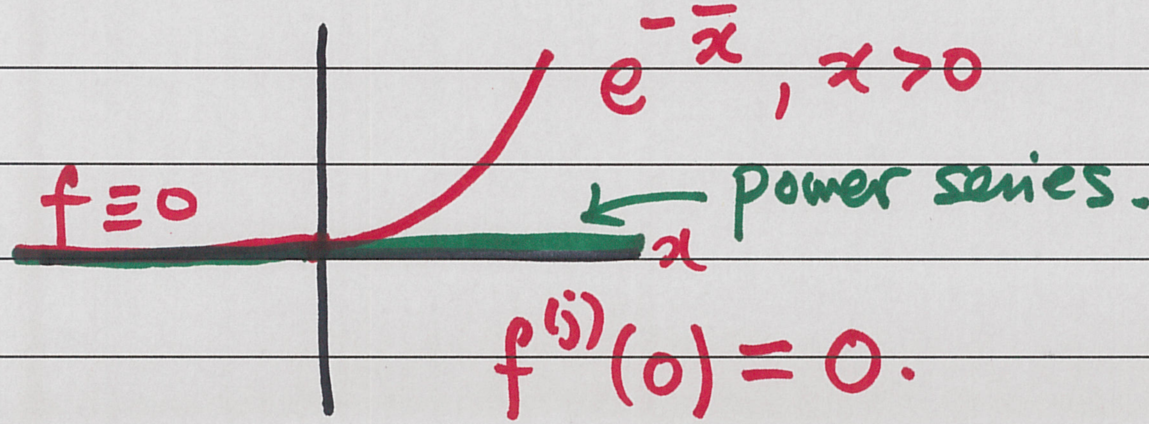
[redacted]

## Power Series

$$e^{x} = \sum_{j=0}^{\infty} \frac{1}{j!} x^{j}$$

In general, $f(x) \sim \sum_{j=0}^{\infty} \frac{f^{(j)}(x_0)}{j!} (x-x_0)^{j}$

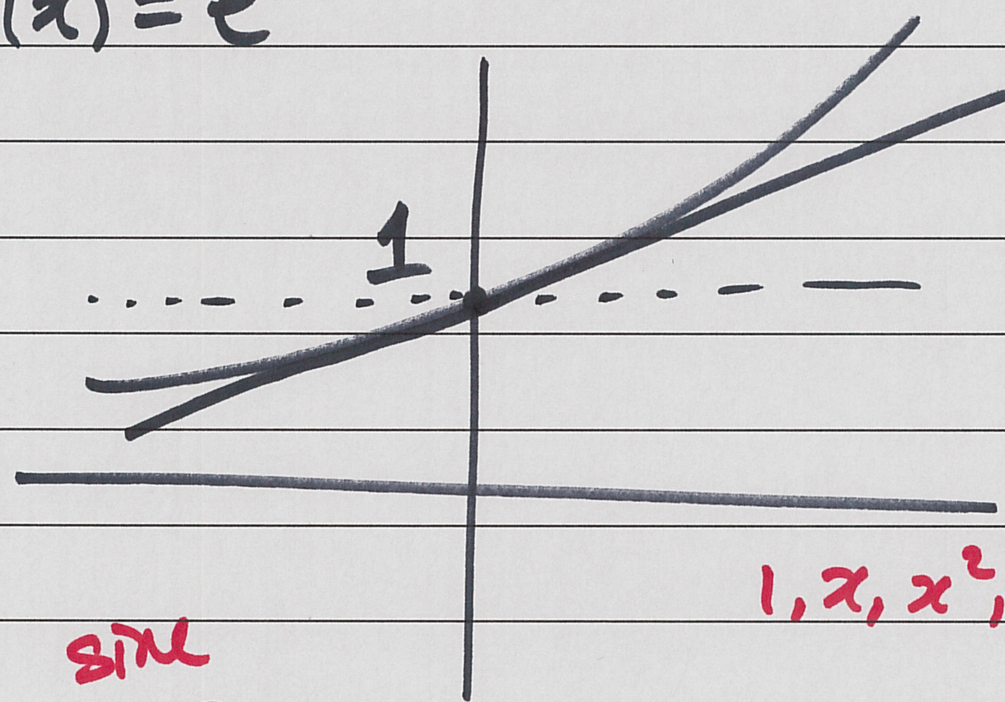
(as long as the derivatives, $f'(x_0), f''(x_0), \ldots, f^{(j)}(x_0), \ldots$ exist.)




$f^{(j)}(0) = 0.$

[redacted]

$f(x) = e^x$




$l(x) = 1 + x$

$q(x) = 1 + x + \frac{1}{2}x^2$

$1, x, x^2, x^3, x^4, \ldots$

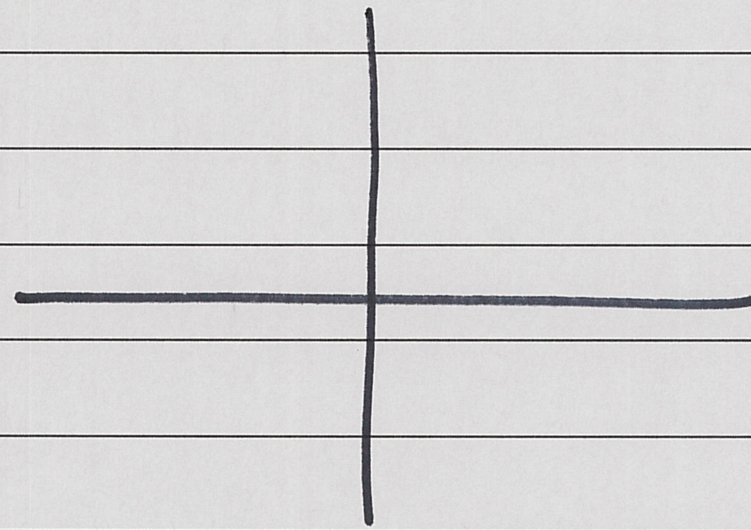
Fourier sine Series: $\sin \omega x, \sin 2\omega x, \sin 3\omega x, \ldots$

Fourier sine basis.

$$\boxed{f(x) = \sum_{j=1}^{\infty} a_j \sin j\omega x}$$

[redacted]

$\underline{e^x}$, <u>Interval</u> of <u>interest</u> $[0, L]$



Come back to this.

[redacted]

$$\frac{d}{dx} \sin(jwx) = jw \cos(jwx)$$

$$\frac{d^2}{dx^2} \sin(jwx) = -(jw)^2 \sin(jwx).$$

---

$$\frac{\partial^2}{\partial x_2^2}\left[\underbrace{\sin(jwx_1) \sin(jwx_2)}\right] =$$

$$-(jw)^2 \underbrace{\sin(jwx_1) \sin(jwx_2)}$$

$$\Delta\left[\underbrace{\sin(jwx_1) \sin(jwx_2) \cdots \sin(jwx_n)}_{B(x_1, \dots, x_n)}\right]$$

$$= -n(jw)^2 B$$

[redacted]

## Fourier

$$u(\mathbf{x},t) = e^{-n(jw)^2 t}\, B(\mathbf{x}) \quad , \quad \mathbf{x} = (x_1, \ldots, x_n)$$

$$\frac{\partial u}{\partial t} = u_t = -n\,(jw)^2\, u$$

$$\left( \Delta B = -n\,(jw)^2\, B \right)$$

The Fourier basis has solutions of the heat equation "built into" it.

$$\Delta u = -n\,(jw)^2\, u = u_t$$

↑
spatial Laplacian

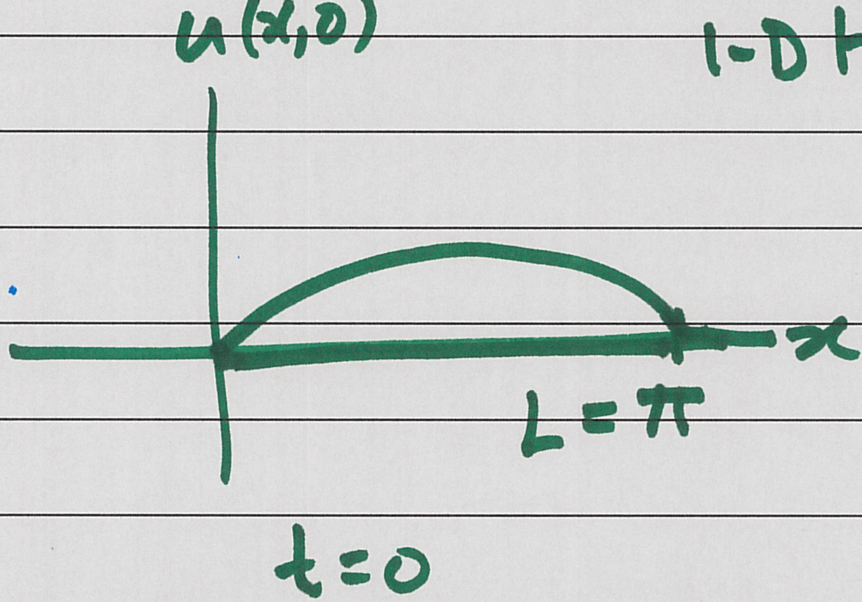
[redacted]

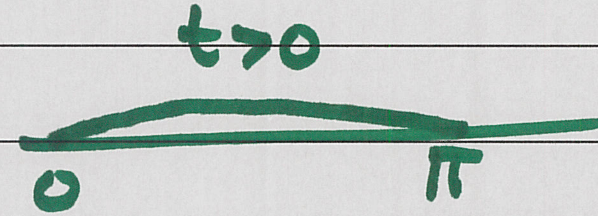
<u>n=1</u>

$$u(x,t) = e^{-t} \sin x$$

<u>satisfies</u> $\boxed{\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}}$

1-D Heat equation.




[redacted]

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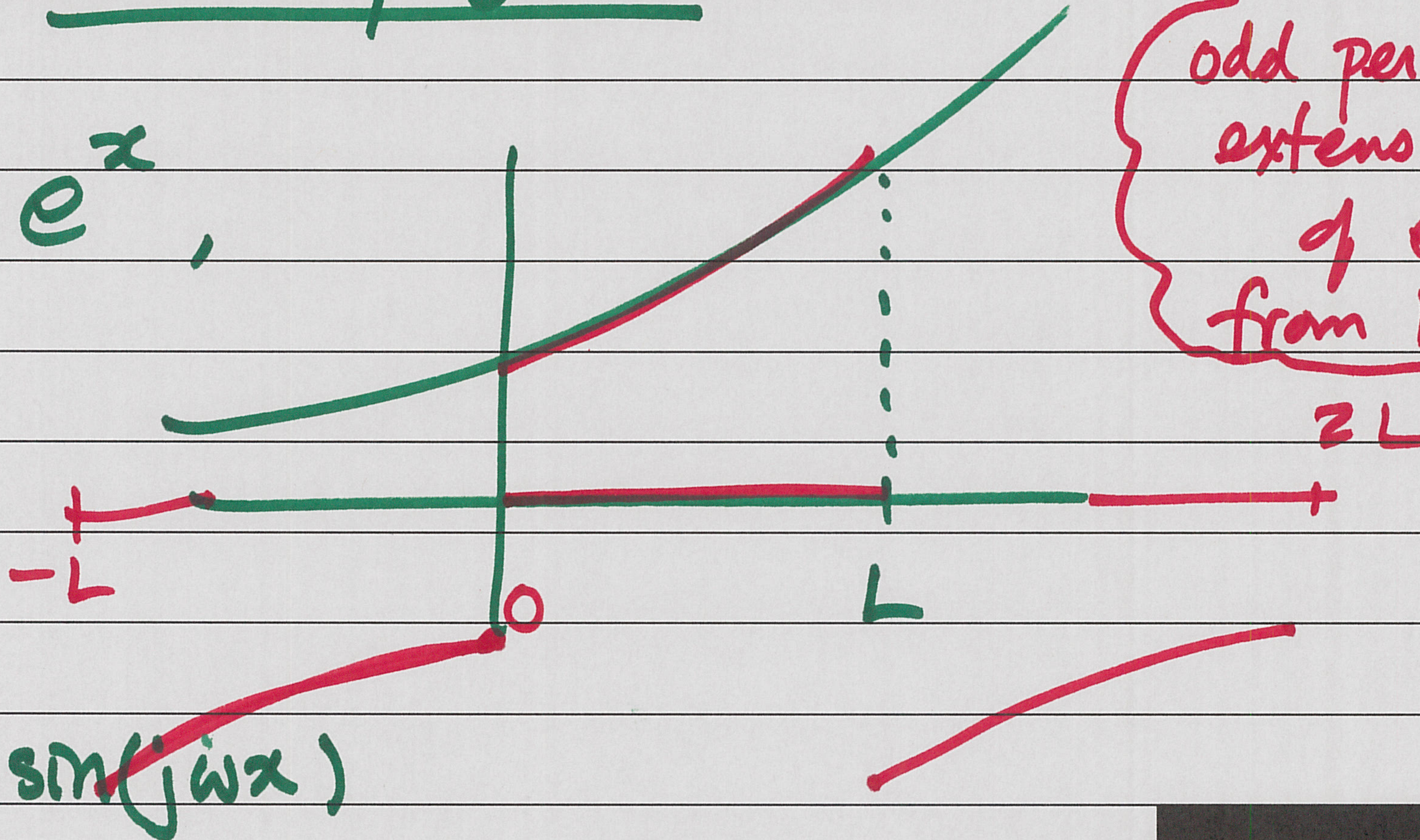
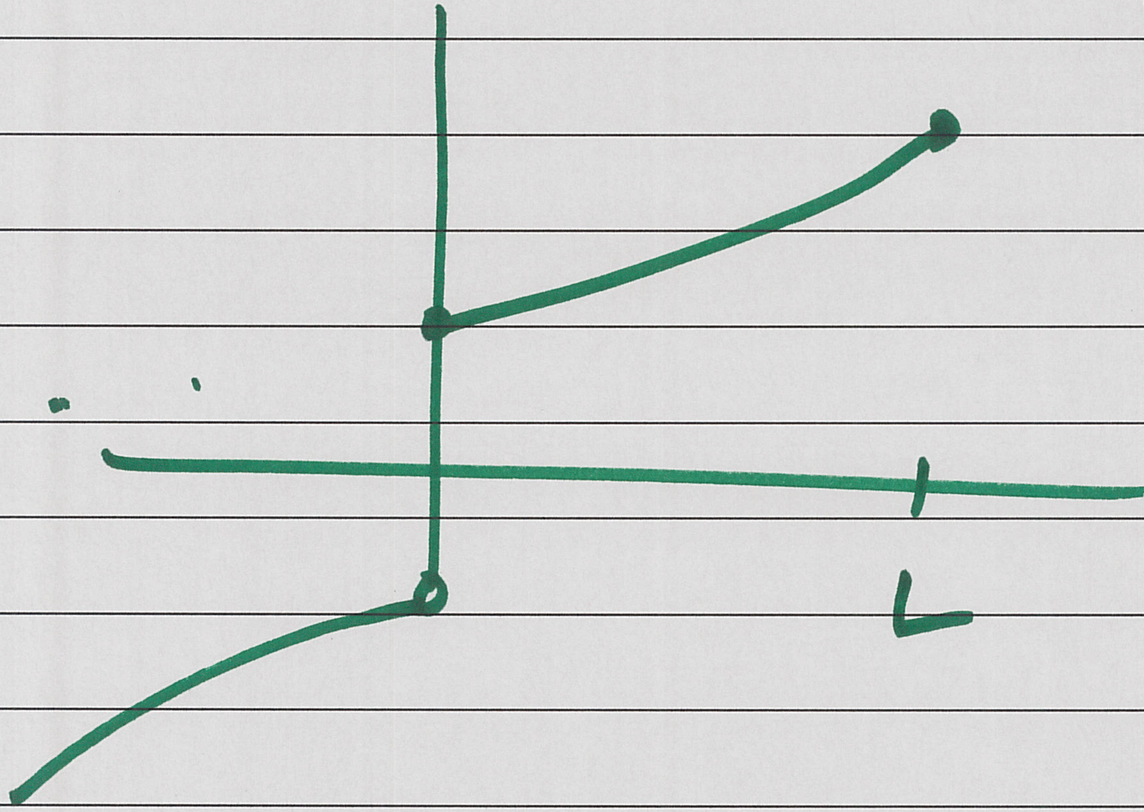



odd periodic extension of $e^x$ from $[0, L]$.

At least we have a chance to represent this extension as a Fourier series.

[redacted]




Pick $\underline{\underline{\omega}}$

$\sin(j\omega x)$

$\omega = 1$ ? . $\omega = L$ ?

$\sin(\omega x)$ | $\sin(Lx)$
$\sin x$ | $\underline{\underline{\frac{2\pi}{L}}}$
$\underline{\underline{2\pi}}$ |

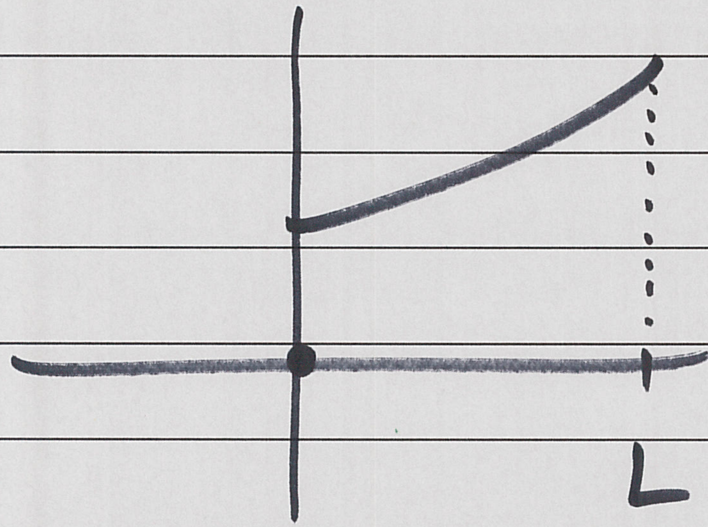
extension has period $\underline{\underline{2L}}$

$\omega = \frac{\pi}{L} \longrightarrow \frac{2\pi}{\omega} = 2L$

$\sin\left(\frac{j\pi}{L} x\right)$

[redacted]

$e^x$ , $\sin\frac{\pi x}{L}$, $\sin\frac{2\pi x}{L}$, $\sin\frac{3\pi x}{L}$, ...




odd periodic extension

$\underline{\underline{g(x)}}$

$$\boxed{g(x) \sim \sum_{j=1}^{\infty} a_j \sin\frac{j\pi x}{L}}$$

<u>Step 3</u> : Find the coefficients $a_j$.

[redacted]

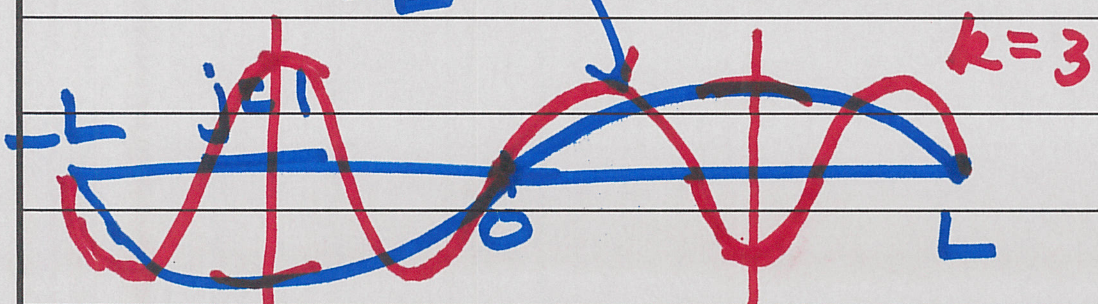
## Step 3

$$\boxed{g(x) = \sum_{j=1}^{\infty} a_j \sin \frac{j\pi x}{L}}$$

$$\int_{-L}^{L} g(x) \sin \frac{k\pi x}{L}\, dx = \sum_{j=1}^{\infty} a_j \int_{-L}^{L} \sin \frac{j\pi x}{L} \sin \frac{k\pi x}{L}\, dx$$

$$\int_{-L}^{L} \sin \frac{j\pi x}{L} \sin \frac{k\pi x}{L}\, dx = 0 \quad j \neq k$$

(believe this for now)



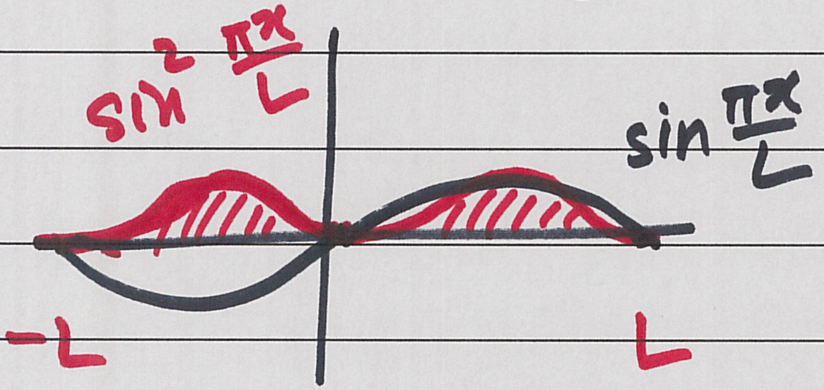

[redacted]

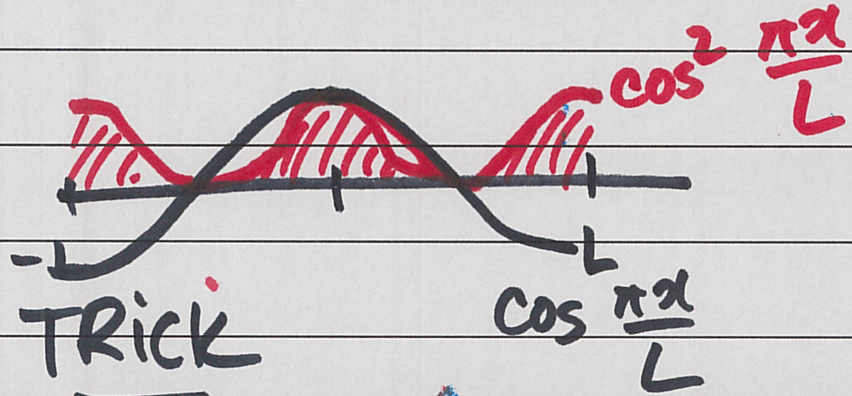
<u>$k=j$</u>

$$\int_{-L}^{L} \sin^2\left(\frac{k\pi x}{L}\right) dx > 0.$$

$$= \int_{-L}^{L} \cos^2\left(\frac{k\pi x}{L}\right) dx$$

$$\boxed{2L = 2\int_{-L}^{L} \sin^2\left(\frac{k\pi x}{L}\right) dx}$$




<u>TRICK</u>

[redacted]

$$\int_{-L}^{L} g(x)\sin\frac{k\pi x}{L}\,dx = \sum_{j=1}^{\infty} a_j \underbrace{\int_{-L}^{L} \sin\frac{j\pi x}{L}\sin\frac{k\pi x}{L}\,dx}$$

$$= a_k \cdot L$$

$$a_k = \frac{1}{L}\int_{-L}^{L} g(x)\sin\frac{k\pi x}{L}\,dx$$

$$= \frac{2}{L}\int_{0}^{L} e^{x}\sin\frac{k\pi x}{L}\,dx$$

[redacted]