MATH 4581

Classical Mathematical Methods in Engineering.

Tuesday August 24, 2021

<u>Intro lecture</u> <u>John</u> McCuan

http://math.gatech.edu/courses/4581/

- talk to me
- ask questions

[redacted]

Familiar (?)	4581
Ordinary Differential Eqns	Partial Differential Eqns.
Power Series	Fourier Series

[redacted]

Ordinary Differential Eqns (ODE)

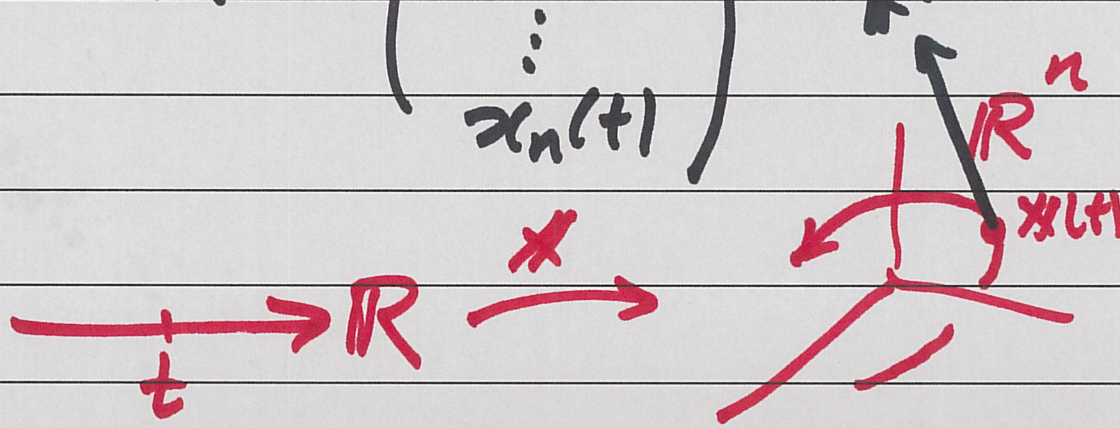
$$\boxed{\mathbb{X}'(t) = \mathbb{F}(\mathbb{X}(t), t)}$$

$\mathbb{F} : \mathbb{R}^n \times \mathbb{R} \longrightarrow \mathbb{R}^n$

1^{st} order system of ODEs.

$$\mathbb{X} = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

$\mathbb{X} : \mathbb{R} \longrightarrow \mathbb{R}^n$



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Every ODE is (equivalent to)

$$\boxed{\mathbb{X}' = \mathbb{F}(\underline{\underline{\mathbb{X}}}, t)}$$

$$\underline{y'' + 3y' + y} = 7$$

linear operator

$$L[y] = y'' + 3y' + y$$

$$\begin{cases} x_1 = y \\ x_2 = y' \end{cases}, \quad \mathbb{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\mathbb{X}' = \begin{pmatrix} x_2 \\ 7 - x_1 - 3x_2 \end{pmatrix}$$

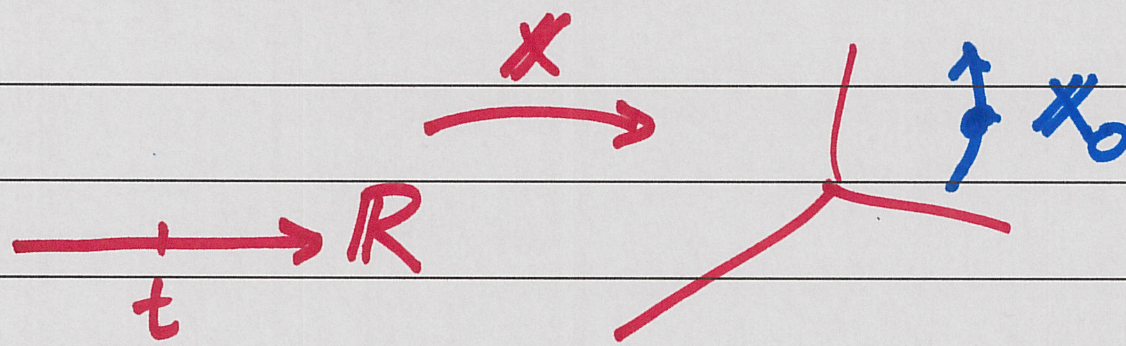
$$\mathbb{X}' = \begin{pmatrix} 0 & 1 \\ -1 & -3 \end{pmatrix} \mathbb{X} + \begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

[redacted]

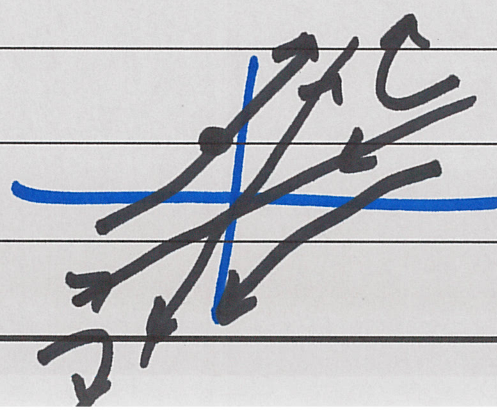
1. $\mathbb{X}' = \mathbb{F}(\mathbb{X}, t)$

There is a general theory of

2. $\boxed{\text{Existence And Uniqueness}}$ (of solutions)



Remember:



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There is no general theory of

Partial Differential Equations (PDE)

The <u>three</u> intro (linear constant coeff. 2nd order) PDE's

Laplace's Equation $\quad \dfrac{\partial^2 u}{\partial x_1^2} + \dfrac{\partial^2 u}{\partial x_2^2} + \cdots + \dfrac{\partial^2 u}{\partial x_n^2} = 0$

(ELLIPTIC)

Here: $\quad u : \mathbb{R}^n \to \mathbb{R}, \quad u = u(x_1, x_2, \ldots, x_n)$

Heat Equation : $\quad \dfrac{\partial u}{\partial t} = \Delta u$

$u : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ (PARABOLIC)

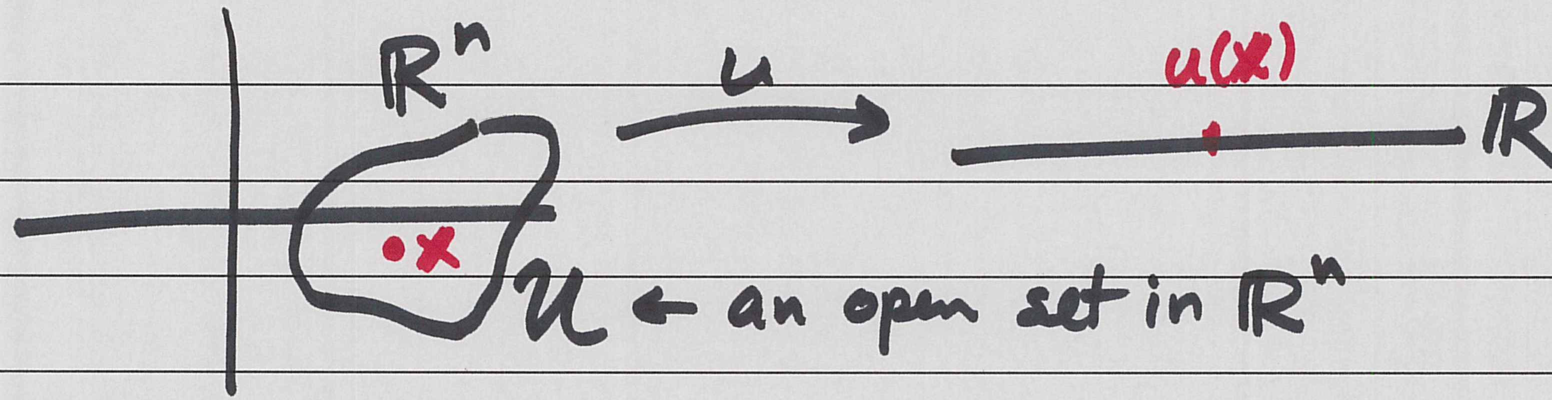
WAVE Equation : $\quad \dfrac{\partial^2 u}{\partial t^2} = \Delta u$

(hyperbolic)

[redacted]

$u : \cancel{\mathbb{R}^n} \longrightarrow \mathbb{R}$

$\mathcal{U} \subseteq \mathbb{R}^n$



- <u>GRAPHS</u>
- ∇ and Δ
- textbook
  <u>Haberman</u>

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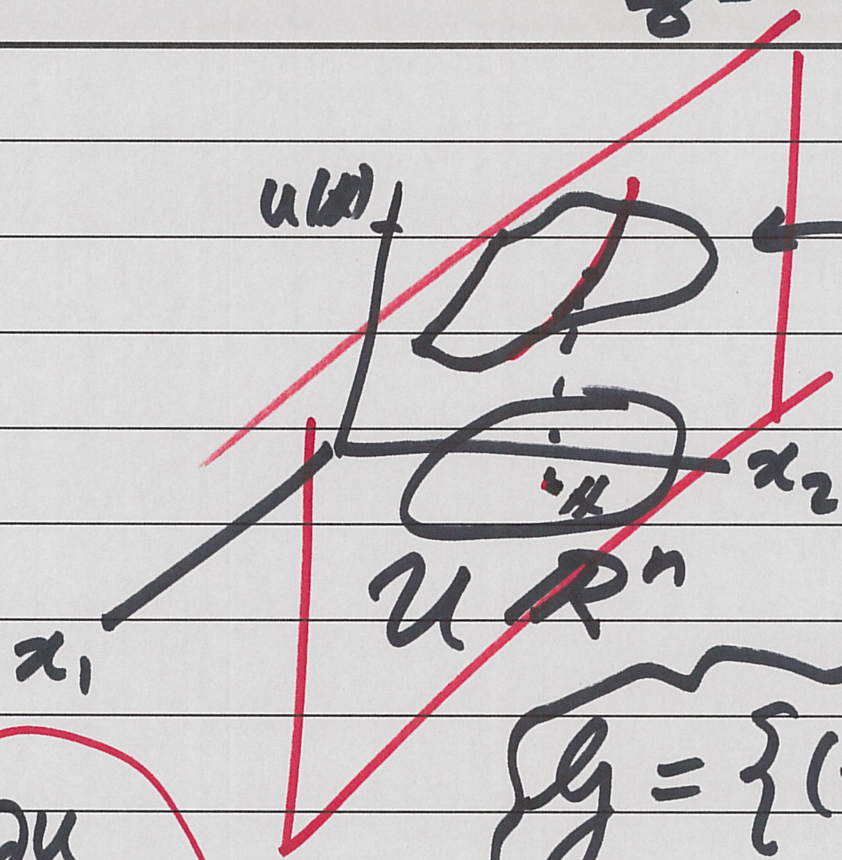


← graph of u

(a surface)

in $\mathbb{R}^{n+1}$

$$\mathcal{G} = \{ (x_1, x_2, u(x_1, x_2)) : (x_1, x_2) \in U \} \subset \mathbb{R}^3$$

"The set of" "such that"

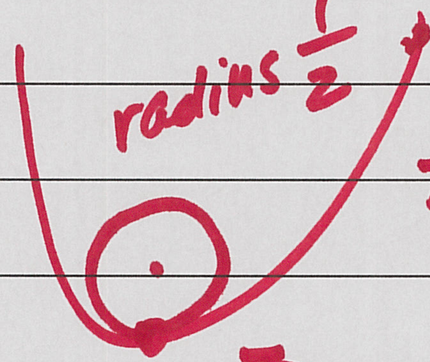
$$\frac{\partial u}{\partial x_1}$$

$$\frac{f''(x)}{[1 + f'(x)^2]^{3/2}}$$

$\vec{x}$ or $\underline{x}$ or $\mathbb{x}$

vectors

[redacted]



$f(x) = x^2$

$$\frac{f''(x)}{\left[1+(f'(x))^2\right]^{3/2}}$$

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = 0$$

[redacted]

$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_n}\right)$$

"nabla"

$$\nabla u = \left(\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \cdots, \frac{\partial u}{\partial x_n}\right)$$

The gradient (total derivative)

Du

$$\nabla \cdot \mathbb{V} = \frac{\partial u}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \cdots + \frac{\partial v_n}{\partial x_n} = \sum_{j=1}^{n} \frac{\partial v_i}{\partial x_j}$$

$\mathbb{V} = (v_1, v_2, \ldots, v_n)$ divergence

$\operatorname{div} \mathbb{V}$

$$\nabla^2 u = \nabla \cdot \nabla u = \sum_{j=1}^{n} \frac{\partial^2 u}{\partial x_j^2}$$

Δu

[redacted]

The Book

Haberman Intro to PDE

[redacted]

MATH 4581 Lecture 2

Thursday Aug. 26, 2021

- Review of Last time
- Review Power Series -versus- Fourier Ser.
- 1-D Heat Eqn.

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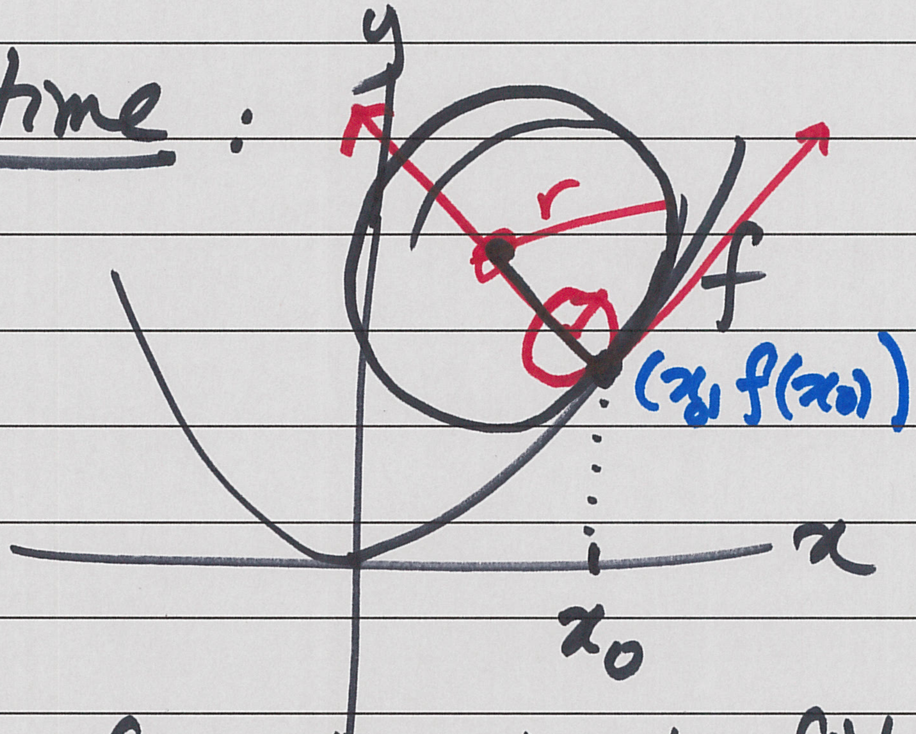
Last time :



$$\frac{(1, f'(x_0))}{\sqrt{1+f'(x_0)^2}}$$

Can you find the best fitting circle to the graph of $y = f(x)$ at $(x_0, f(x_0))$?

(best fitting ??)

(i) $(x_0, f(x_0))$ is on the circle.

(ii) circle tangent to graph

⋮ $f'(x_0) = g'(x_0)$, $f(x_0) = g(x_0)$

[redacted]