MATH 4581 Thursday December 2, 2021

Lecture 28

- Problem 2 Assignment 7

office hours

u_0 odd and $2L$ periodic

$$\Rightarrow \begin{cases} u_0(0) = 0 \\ u_0(L) = 0 \end{cases} \longrightarrow \begin{array}{l} u(0,t) = 0 \\ u(L,t) = 0 \end{array}$$

v_0 too

does not use continuity

de Alembert formula

$$\left.\begin{aligned} u_0(L) &= u_0(L-2L) \\ &= u_0(-L) \\ &= -u_0(L) \end{aligned}\right\} \Rightarrow 2u_0(L) = 0$$

[redacted]

Assignment 7 Problem 1 (c).

$$
\begin{cases}
u_{tt} = \sigma^2 u_{xx} & \text{on } \underline{[0,L]} \times [0,\infty) \\
u(0,t) = 0 = u(L,t) \\
u(x,0) = \underset{\substack{\uparrow \\ g}}{\cancel{X_0}}, \quad u_t(x,0) = \underset{\substack{\uparrow \\ h}}{\cancel{V_0}}
\end{cases}
$$

$g, h \in C^1[0,L]$

<u>d'Alembert</u> ?

problem: x fixed, t big means $x \mp \sigma t \notin [0,L]$

NO?

$$u(x,t) = \frac{1}{2}\left[g(x-\sigma t) + g(x+\sigma t)\right] + \frac{1}{2}\int_{x-\sigma t}^{x+\sigma t} h(\xi)\, d\xi$$

[redacted]

<u>IDEA</u>: Extend g and h to all of $\mathbb{R}$.

- extend them to be odd and $2L$ periodic.

- continuous?

$$u(x,t) = \frac{1}{2}\left[\bar{g}(x-\sigma t) + \bar{g}(x+\sigma t)\right] + \frac{1}{2}\int_{x-\sigma t}^{x+\sigma t} \bar{h}(\xi)\, d\xi$$

<u>MAKES SENSE</u>.

[redacted]

<u>$x \neq 0$</u>

$$\frac{1}{\sqrt{4\pi}} \lim_{t \to 0} \frac{e^{-\frac{x^2}{4t}}}{\sqrt{t}} = \frac{0}{0}$$ indeterminate form

→ L'Hopital's rule

$$= \frac{1}{\sqrt{4\pi}} \lim_{t \to 0} \sqrt{\frac{e^{-\frac{x^2}{2t}}}{t}} \to 0.$$

<u>look</u> at $\lim_{t \to 0} \frac{1}{t e^{\frac{x^2}{2t}}}$ $\to +\infty$

look at $\lim_{t \to 0} t e^{\frac{x^2}{2t}} = 0 \cdot \infty$

[redacted]

$$\lim_{t \searrow 0} t\, e^{\frac{x^2}{2t}} = 0 \cdot \infty \text{ indeterminate.}$$

$$\lim_{R \nearrow \infty} \frac{1}{R} e^{R \frac{x^2}{2}} = \lim_{R \nearrow \infty} \frac{e^{R\left(\frac{x^2}{2}\right)}}{R} = \frac{\infty}{\infty}$$

$$\frac{d}{dt} e^{\frac{x^2}{2t}} = e^{\frac{x^2}{2t}} \left(-\frac{x^2}{2t^2}\right)$$

$$\uparrow R\left(\tfrac{x^2}{2}\right)$$

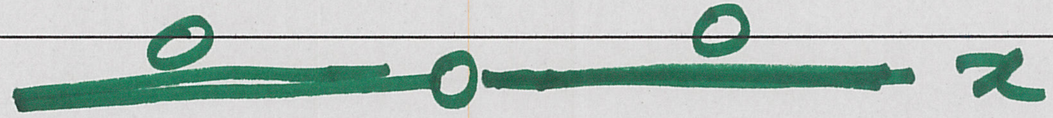
$$\lim_{R \nearrow \infty} \frac{e^{R\left(\frac{x^2}{2}\right)}}{R} \sim \lim_{R \nearrow \infty} \frac{\frac{x^2}{2} e^{R\left(\frac{x^2}{2}\right)}}{1}$$

$$= +\infty$$

[redacted]

α 7

$\lim_{t \to \infty} \Phi(x,t)$



Dirac Delta distribution

$$\int_{\mathbb{R}} f(x)\,\delta(x) = \begin{cases} \cancel{f(0) \quad (\text{engineer})} \\ 0 \quad (\text{mathematician}) \end{cases}$$

$$\delta : C^0(\mathbb{R}) \to \mathbb{R}$$

$$\delta[f] = f(0).$$

linear functional

[redacted]