MATH 4581 Lecture 23 Thursday November 11, 2021

- Streamlines: Problem 10 Assignment 5

- Sturm-Liouville Theory and Bessel Functions
  Ch. 5 Ch. 7

Last time: $\psi(r,\theta) = -\alpha \ln r - \left(r - \frac{1}{r}\right)\sin\theta$

$$F(x,y) = -\alpha \ln\sqrt{x^2+y^2} - \left(1 - \frac{1}{x^2+y^2}\right)y$$

$$\Delta\psi = 0, \quad \vec{v} = (\psi_y, -\psi_x)$$

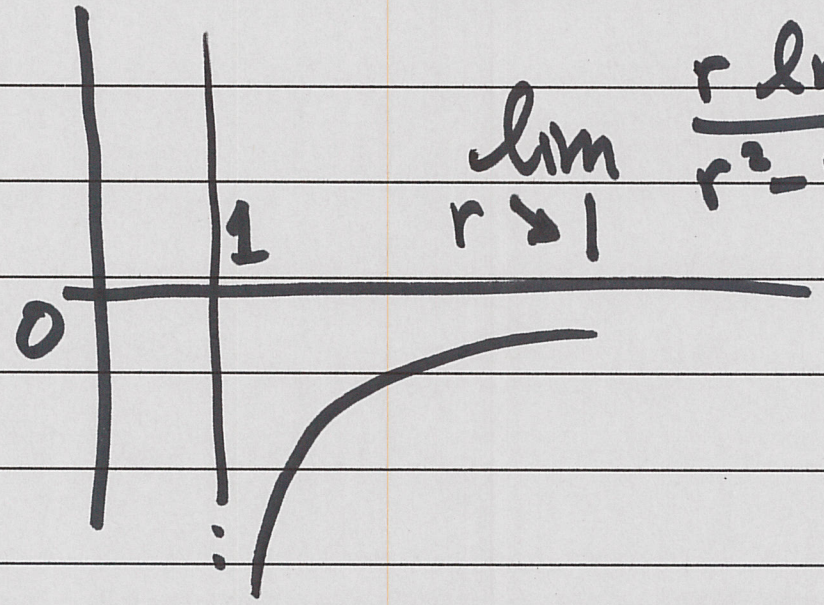
$$\operatorname{div}\vec{v} = (\psi_y)_x + (-\psi_x)_y$$
$$= 0$$

Zero level set / curve of

$$\psi(r,\theta) = -\alpha \ln r - \left(r - \frac{1}{r}\right)\sin\theta$$

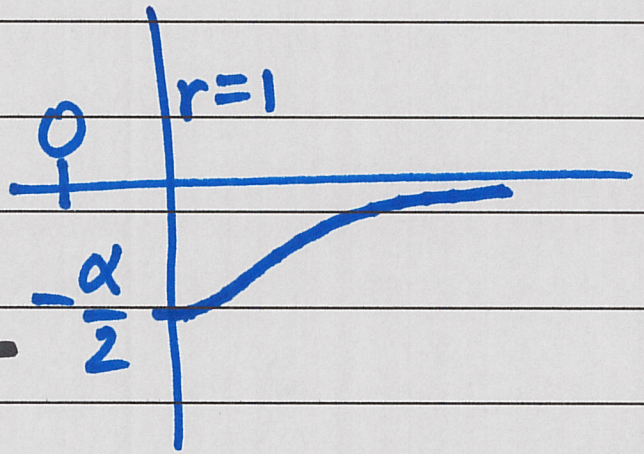
$$\sin\theta = -\frac{\alpha \ln r}{r - \frac{1}{r}} = -\frac{\alpha\, r \ln r}{r^2 - 1}$$

$$\lim_{r \searrow 1} \frac{r \ln r}{r^2 - 1} = \frac{0}{0}$$



$$\lim_{r \to 1} \frac{r \ln r}{r^2 - 1} = 0.5$$

$$\lim_{r \to 1} \frac{\ln r + 1}{2r} = \frac{1}{2}$$



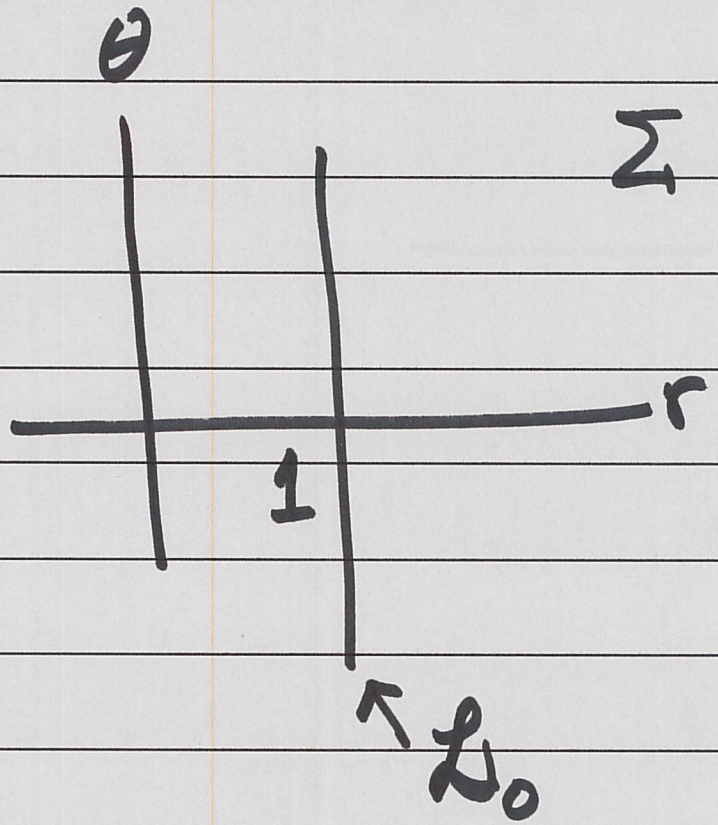
$$\lim_{r \to 1} \sin\theta = \lim_{r \to 0^-} -\frac{\alpha r \ln r}{r^2 - 1} = -\frac{\alpha}{2}$$

$\alpha = 1/2$

$$-\frac{1}{4}$$

$$\sin\theta = \frac{-\alpha r \ln r}{r^2 - 1}$$

looking for values in $[-1, 0)$



$$\Sigma = \{(r,\theta) : r \geq 1\}$$

$$\theta = -\sin^{-1}\left(\frac{\alpha r \ln r}{r^2-1}\right)$$

$$\Psi(r,\theta) = -\alpha \ln r - \left(r - \frac{1}{r}\right)\sin\theta \stackrel{?}{\equiv} 0.$$

$$\sin\theta = -\frac{\alpha r \ln r}{r^2-1}$$