Math 4431, Assignment 8a: Chapter 4 Name and section: ____________

§ 4.1-2 Armstrong

Definition/Proposition If X is a topological space, A is a set, and $p : X \twoheadrightarrow A$ is a surjective function, then p is called a (generalized) **projection** of X onto A, and A becomes a topological space with topology

$$\mathcal{Q} = \{V \subset A : p^{-1}(V) \text{ is open in } X\}.$$

This topology is called the **quotient topology** on A.

1. (10 points) Figure out what needs to be proved as the "proposition" part of the definition/proposition above, and prove it.

2. (10 points) Consider

   $$\mathcal{P}_M = \{\{(x,y)\} : 0 < x < 2\pi,\ -1 \leq 2y \leq 1\} \cup \{\{(0,y),(2\pi,-y)\} : -1 \leq 2y \leq 1\}.$$

   This is a partition of the rectangle $X = [0, 2\pi] \times [-1/2, 1/2]$. Show the function $p : X \to \mathcal{P}_M$ by $p(x) = P$ where $x \in P$ is well-defined and onto. What is the quotient space $\mathcal{P}_M$?

3. (10 points) Show that the projection map $p : X \to \mathcal{P}_M$ in the last problem is not a homeomorphism.

4. (10 points) Show that the quotient topology induced on Y by a homeomorphism $h : X \to Y$ is the same as the topology on Y. Can you show that the rectangle X (from problem 2) is not homeomorphic to $\mathcal{P}_M$?

5. (10 points) Show that a continuous surjective function $q : X \to Y$ which is **closed**, i.e., $q(A)$ is closed for every closed set $A \subset X$, is an **identification map**.