Math 4431, Assignment 7: Chapter 3 Name and section: ____________________

§ 3.3-6 Armstrong

Definition A topological space X is **locally connected** if each point $x \in X$ and each open set U with $x \in U$, there is an open connected set V with $x \in V \subset U$.

1. (20 points) (3.5.34) Show that if X is homeomorphic to an open set in $\mathbb{R}^n$, then X is locally connected.

2. (20 points) (3.5.34) Show that $X = \{1/j \in \mathbb{R} : j = 1, 2, 3, \ldots\}$ is locally connected (as a subset/subspace of $\mathbb{R}$), but $X \cup \{0\}$ is not.

3. (20 points) (3.6.37-38) Show that $\mathbb{S}^2$ is path connected.

4. (20 points) (3.6.41) Let X be a locally connected topological space. If $A \subset X$ is path connected, then can you show $\overline{A}$ is connected? Can you show $\overline{A}$ is path connected?

5. (20 points) Let $U \subset \mathbb{R}^2$ be open and connected with $x_0 \in U$. Let Γ denote the family of all continuous paths $\gamma : [0, 1] \to U$ with $\gamma(0) = x_0$, and let $\{\gamma\}$ denote the image of γ, i.e.,

   $$\{\gamma(t) : t \in [0, 1]\}.$$

   Show

   $$\cup_{\gamma \in \Gamma} \{\gamma\}$$

   is closed.