Math 4431, Assignment 6b: Chapter 3 Name and section: ________________

§ 3.3-5 Armstrong

Definition A topological space X is **locally compact** if each point $x \in X$ has a compact neighborhood, i.e., if there is a compact set N and an open set U with $x \in U \subset N$.

1. (10 points) (3.3.15) Show that any compact space is locally compact.

2. (10 points) (3.3.17 important) Let X be a locally compact Hausdorff space (like $\mathbb{R}^n$). Show there is a compact topological space

$$\tilde{X} = X \cup \{\infty\}$$

where "∞" denotes a single additional point (not in X) and $\tilde{X}$ has open sets

$$\tilde{\mathcal{T}} = \{U : U \text{ is open in } X\} \cup \{(X \backslash K) \cup \{\infty\} : K \subset X \text{ is compact}\}.$$

$(\tilde{X}, \tilde{\mathcal{T}})$ is called the **one point compactification** of X.

3. (10 points) (3.4.20) Let X and Y be topological spaces. If $A \subset X$ and $B \subset Y$, then $\overline{A \times B} = \overline{A} \times \overline{B}$.

4. (10 points) (3.5.31) Let X be the real line with the finite complement topology. If $x \in X$, then what is the (unique) component containing x? What does this tell you about X?

5. (10 points) (3.5.32) If $X = \cup_{j=1}^k C_j$ where $C_1, \ldots, C_k$ are the distinct components of X, then show that C_j is open for $j = 1, \ldots, k$.