Math 4431, Assignment 3: metric topology Name and section: ____________________

§ 2.4 Armstrong; X is a metric space.

1. (20 points) (2.4.27) If $A \subset X$ and $x \in \bar{A}$, then $d(x, A) = 0$.

2. (20 points) (2.4.28) Show that if A_1 and A_2 are disjoint closed sets (in the metric space X), then there are disjoint open sets U_1 and U_2 with $A_j \subset U_j$ for $j = 1, 2$.

3. (20 points) (2.4.31) Give an example of a metric space X containing nonempty disjoint closed sets A_1 and A_2 such that

$$\inf_{x \in X} [d(x, A_1) + d(x, A_2)] = 0.$$

4. (20 points) If $M > 0$ and $0 < r < 1$ and for each $j = 1, 2, 3, \ldots$ we have $f_j \in C(X)$ with

$$\sup_{x \in X} |f_j(x)| \leq M r^j,$$

then show

$$\bar{f}(x) = \sum_{j=1}^{\infty} f_j(x) = \lim_{k \to \infty} \sum_{j=1}^{k} f_j(x)$$

is well-defined for each $x \in X$ and $\bar{f} \in C(X)$.

5. (20 points) (2.4.33) Let $A = \mathbb{R} \backslash \{0\} \subset \mathbb{R}$. Find a function $f \in C^0(A)$ which cannot be extended to $X = \mathbb{R}$.