Math 4431, Assignment 11a: Chapter 5 Name and section: ________________

§ 5.1-2 Armstrong: The Fundamental Group

Read § 1.5-6 of Chapter 1 in Armstrong.

Let X be a path connected topological space.

A **loop** in X is a continuous function $\gamma : [0,1] \to X$ with $\gamma(0) = \gamma(1)$. The set of all loops γ with $\gamma(0) = \gamma(1) = p \in X$ is called the **loop space of** X **at** p. Denote the loop space at p by Λ_p.

Two loops γ_0 and γ_1 in Λ_p are said to be **homotopic** if there is a continuous function

$$H : [0,1] \times [0,1] \to X$$

such that

(i) $H(t,0) \equiv \gamma_0(t)$,

(i) $H(0,s) \equiv H(1,s) \equiv p$ for $0 \le s \le 1$, and

(i) $H(t,1) \equiv \gamma_1(t)$.

Note that for each fixed s, the function $\eta : [0,1] \to X$ by $\eta(t) = H(t,s)$ is a loop in Λ_p. Sometimes we will write $\eta(t) = \eta_s(t) = \eta(t;s) = H(t,s)$.

1. (10 points) (1.6.14,26) Make a Möbius strip from paper and cut it in half along the center circle. Show the result is homeomorphic to a cylinder. Can you manipulate a paper cylinder into a Móbius strip by identifying opposite points on one boundary circle?

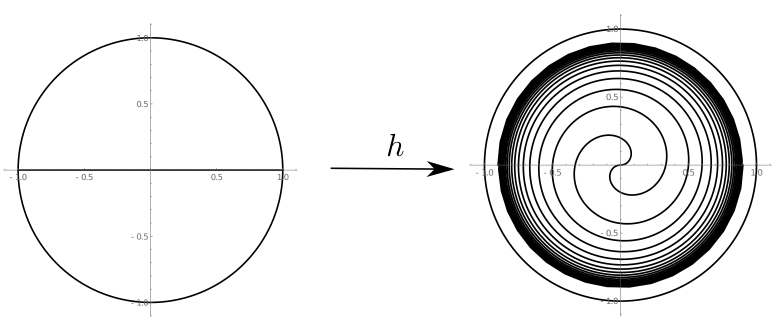
2. (10 points) (1.6.23) Let $X = \mathbb{S}^1 \cup \{(x,0) : 0 \le x \le 1\} \subset \mathbb{R}^2$. Show that X and $\mathbb{S}^1 \subset \mathbb{R}^2$ are not homeomorphic.

3. (10 points) Let $p, q \in B_1(0) \subset \mathbb{R}^2$. Find an homeomorphism $h : \overline{B_1(0)} \to \overline{B_1(0)}$ with $h(p) = q$ and $h(q) = p$.

4. (10 points) Again consider $B_1(0) \subset \mathbb{R}^2$ and the function $h : B_1(0) \to B_1(0)$ by

$$h(x,y) = r(\cos(\theta + 2\pi r/(1-r)), \sin(\theta + 2\pi r/(1-r)))$$

where $r = \sqrt{x^2 + y^2}$ and $(x,y) = r(\cos\theta, \sin\theta)$ with $0 \le \theta < 2\pi$. Is h a homeomorphism? Justify your answer.



5. (10 points) Let $\overline{h} : \overline{B_1(0)} \to \overline{B_1(0)}$ be an extension of the function in the previous problem. Is $\overline{h}$ continuous? Jusfify your answer.

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6. (10 points) If $f : [0,1] \to [0,1]$ is any increasing continuous function with $f(0) = 0$ and $f(1) = 1$ and $\gamma : [0,1] \to X$ is any loop in $\Lambda_p = \Lambda_p(X)$, then show $\alpha : [0,1] \to X$ by $\alpha(t) = \gamma \circ f(t)$ is homotopic to γ.