Assignment 9: Elementary functions
Chapter 3 of BC
Due Wednesday, March 22, 2023

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Problem 1 (Exercise 3.30.8 in BC) Find all complex numbers $z \in \mathbb{C}$ for which

(a) $e^z = -2$.

(b) $e^z = 0$.

(10 presentation points)

Problem 2 (Exercise 3.30.10 in BC) What can you say about $z \in \mathbb{C}$ if $e^z \in i\mathbb{R}$?
(10 presentation points)

Problem 3 (Exercise 3.33.3 in BC) Characterize the integers j for which

$$\log_j(i^3) = 3\log_j(i).$$

(10 presentation points)

Problem 4 (Exercise 3.33.4 in BC) Characterize the integers j for which

$$\log_j(i^2) = 2\log_j(i).$$

(10 presentation points)

Problem 5 (Exercise 3.33.5 in BC) Characterize the integers $j \in \mathbb{Z}$ and $k \in \{0, 1\}$ for which

$$\log_j(\mathrm{sqrt}_k(i)) = \frac{1}{2}\log_j(i)$$

where $\log_j : \mathcal{L}_j \to \Sigma_j$ as usual and $\mathrm{sqrt}_k : \mathcal{R}_k \to \mathbb{C}$ denotes the k branch of the complex square root on the two sheeted Riemann surface $\mathcal{R} = \mathcal{R}_0 \cup \mathcal{R}_1$ associated with z^2.
(10 presentation points)

Problem 6 (Exercise 3.33.7 in BC) Calculate

$$\frac{d}{dz}\log_j(z) \qquad \text{for } z \in \mathrm{int}(\mathcal{L}_j),\ j \in \mathbb{Z}.$$

(10 presentation points)

Problem 7 (Exercise 3.33.8 in BC) Find all $z \in \mathbb{C}$ for which

$$\log z = \frac{\pi}{2}\, i.$$

(10 presentation points)

Problem 8 (Exercise 3.34.1 in BC) Discuss the "identity"

$$\log_j(zw) = \log_k(z) + \log_\ell(w) + 2\pi m i.$$

(10 presentation points)

Problem 9 (Exercises 3.34.4 in BC) Discuss the "identity"

$$z^{1/n} = \exp\left(\frac{\log_j(z)}{n}\right)$$

for $j \in \mathbb{Z}$ and $n \in \mathbb{N} = \{1, 2, 3, \ldots\}$.
(10 presentation points)

Problem 10 (Exercise 3.33.5 in BC) Discuss the "identity"

$$\log_j\left(z^{1/n}\right) = \frac{\ln|z|}{n} + i\left(\frac{\arg(z) + 2(\ell n + k)\pi}{n}\right)$$

for $j, \ell \in \mathbb{Z}$, $n \in \mathbb{N} = \{1, 2, 3, \ldots\}$ and $k \in n = \{0, 1, 2, \ldots, n-1\}$.
(10 presentation points)