Assignment 12: Complex Integration
Chapter 4 of BC
Due Wednesday, April 12, 2023

John McCuan

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Problem 1 (Exercise 4.42.2 in BC) Compute

$$\int_1^2 \left(\frac{1}{t} - i\right)^2 dt.$$

(10 presentation points)

Problem 2 (Exercise 4.2 in my notes and Exercise 4.43.2 in BC) Parameterize $\partial B_r(z_0)$ by argument θ based at z_0 and reparameterize by arclength.
(10 presentation points)

Problem 3 (Exercise 4.3 in my notes and Exercise 4.43.3 in BC) Parameterize the straight line segment from $z_0 = x_0 + iy_0$ to $z_1 = x_1 + iy_1$ with $\alpha(j) = z_j$ for $j = 0, 1$. Reparameterize by arclength.
(10 presentation points)

Problem 4 (Exercise 3.43.4 in BC) State and prove a chain rule for the composition $\alpha \circ \phi$ where $\alpha : [a, b] \to \mathbb{C}$ parameterizes a curve and $\phi \in C^1[c, d]$ is a change of variable.
(10 presentation points)

Problem 5 (Exercise 3.43.5 in BC) State and prove a chain rule for the composition $f \circ \alpha$ where $\alpha : [a, b] \to U$ parameterizes a curve in an open set $U \subset \mathbb{C}$ and $f : U \to \mathbb{C}$ is complex differentiable.
(10 presentation points)

Problem 6 (Exercise 4.6 in my notes) Compute

$$\int_\Gamma \sqrt{w}$$

where $\gamma : [0, \pi] \to \mathcal{R}$ by $\gamma(t) = 3e^{it} \in \mathcal{R}$ and $\mathcal{R} = \mathcal{R}_0 \cup \mathcal{R}_1$ is the two sheeted Riemann surface for z^2.
(10 presentation points)

Problem 7 (4.46.1 in BC) Compute the complex integral

$$\int_\Gamma f$$

where $\gamma : [0, \pi] \to \mathbb{C}$ by $\gamma(t) = 2e^{it}$ and

$$f(z) = \frac{z+2}{2}.$$

(10 presentation points)

Problem 8 (4.46.3 in BC) Compute

$$\int_\Gamma \pi e^{\pi \overline{z}}$$

where

$$\Gamma = \partial\{z \in \mathbb{C} : 0 < \operatorname{Re} z, \operatorname{Im} z < 1\}.$$

(10 presentation points)

Problem 9 (4.46.6 in BC) Compute the complex integral

$$\int_\Gamma f$$

where $\gamma : [0, \pi] \to \mathbb{C}$ by $\gamma(t) = e^{it}$ and

$$f(z) = z^i = \exp(i \log z)$$

is the principal branch, i.e., Brown and Churchill's principal branch, of z^i determined by taking the branch cut for log along the negative real axis and $-\pi < \arg z < \pi$.
(10 presentation points)

Problem 10 (Section 47 in BC) Show that if $\alpha : [a,b] \to \mathbb{C}$ is a continuous complex valued function of one real variable, then

$$\left| \int_a^b \alpha(t)\, dt \right| \leq \int_a^b |\alpha(t)|\, dt.$$

Hint: Follow the argument in the book.
(10 presentation points)