Assignment 10: Elementary functions Chapter 3 of BC Due Wednesday, March 29, 2023

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Problem 1 (Exercises 3.2 and 3.3 in my notes)

(a) Define what it means for an entire function $f : \mathbb{C} \to \mathbb{C}$ to be (complex) periodic.

(b) Find an entire function with period 1 + i.

(10 presentation points)

Problem 2 (Exercise 3.36.1 in BC) Discuss the value(s) of

 $(1+i)^{i}$.

(10 presentation points)

Problem 3 (Exercise 3.36.2 in BC) Discuss the value(s) of

 $(-i)^{i}$.

(10 presentation points)

Problem 4 (Exercise 3.36.3 in BC) Discuss the value(s) of

$$(-1+\sqrt{3} i)^{3/2}$$
.

(10 presentation points)

Problem 5 (Exercise 3.36.5 in BC) How do the branches of $z^{1/n}$ compare/correspond to the branches of

 $e^{\log z/n?}$

(10 presentation points)

Problem 6 (Exercise 3.5 in my notes) How does the exponential function e^z compare/correspond to the branches of

$$e^{z \log e}$$
?

(10 presentation points)

Problem 7 (complex cosine)

- (a) Find formulas for the real and imaginary parts of $f(z) = \cos z$ in terms of x and y in z = x + iy.
- (b) Find the images under cosine of the horizontal and vertical coordinate lines

$$\{x + iy_0 : x \in \mathbb{R}\} \quad \text{and} \quad \{x_0 + iy : y \in \mathbb{R}\}.$$

(c) Find the images under cosine of the rays and circles relative to z = 0

 $\{re^{i\theta_0}: r \in \mathbb{R}\}$ and $\{r_0e^{i\theta}: \theta \in \mathbb{R}\}.$

(d) Find a fundamental domain E for the cosine, i.e., a "nice" subset of \mathbb{C} with

$$\{\cos z : z \in E\} = \mathbb{C}.$$

(10 presentation points)

Problem 8 (complex sine)

- (a) Find formulas for the real and imaginary parts of $f(z) = \sin z$ in terms of x and y in z = x + iy.
- (b) Find the images under sine of the horizontal and vertical coordinate lines

$$\{x + iy_0 : x \in \mathbb{R}\}$$
 and $\{x_0 + iy : y \in \mathbb{R}\}$

(c) Find the images under sine of the rays and circles relative to z = 0

$$\{re^{i\theta_0}: r \in \mathbb{R}\}$$
 and $\{r_0e^{i\theta}: \theta \in \mathbb{R}\}.$

(d) Find a fundamental domain E for the sine, i.e., a "nice" subset of \mathbb{C} with

$$\{\sin z : z \in E\} = \mathbb{C}.$$

(10 presentation points)

Problem 9 (complex cosine) Illustrate the Riemann surface of the complex cosine and define a branch of the complex arccosine. (10 presentation points)

Problem 10 (complex sine) Illustrate the Riemann surface of the complex sine and define a branch of the complex arcsine. (10 presentation points)