

Math 4317, Self Assessment 3

Incompleteness of \mathbb{Q}

Let

$$A = \{x \in \mathbb{Q} : x^2 \leq 2\}$$

where \mathbb{Q} denotes the rational numbers.

1. Show that if $b \in P = \{x \in \mathbb{Q} : x > 0\}$ is positive and $b^2 > 2$ then b is **not** the least upper bound of A , i.e., find a rational number c which is an upper bound for A with $c < b$. Hint: You can use the fact that if $c \in P$ and $c^2 \geq 2$, then c is an upper bound for A . (This was a problem on Self Assessment 2.)

- I was able to do this.
 - I was not able to do this.
 - I see how to do it now.
 - I don't think I'll ever be able to understand this.
 - I refuse to participate in self-assessment.
2. True or False: There exists an element $x \in \mathbb{Q}$ such that $x^2 = 2$.

- I was able to do this.
- I was not able to do this.
 - I see how to do it now.
 - I don't think I'll ever be able to understand this.
- I refuse to participate in self-assessment.