Public Proposal 1 MATH 4317
Evaluation 1
(Identify the problem here.)

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1 Introduction

In MATH 4317 students submit proposals giving solutions (or attempted solutions) of assigned problems. If the proposal is interesting, well-written, and correct, or at least well-written and displays one of the other two characteristics, then the student is allowed to make an in class presentation and gets credit for class participation.

This is an evaluation of a student proposal, submitted anonymously, for instructional purposes. The proposal itself should be available for comparison in full context, though generally when I present an evaluation of some portion of the proposal, I will attempt to reproduce the content under discussion here. Starting in the next section, excerpts will be in blue and comments in red.

There is some red above already which does not constitute an evaluation, but serves as a reminder that it is a good idea to identify the problem in the title and (if you are not submitting the proposal anonymously) put your name on it. In this case, the student mentioned in the email that this was a proposal submitted in connection with Assignment 1A Problem 13.

When you submit proposals for consideration, I will try to meet you at the level where you are and offer some constructive criticism. I am not trying to put you down, but rather help you up to a level of being able to write mathematics clearly. That is what this course is about, and (as explained in the introductory materials) it is not going to be easy. So grow some thick skin and get ready to work hard—harder than you have ever worked in a class before—or else you are wasting your time.

2 Evaluation

Even with the number of the problem stated, say, in the title at the top of the page, an acceptable presentation should include a detailed description of the problem with appropriate definitions and a discussion of concepts which may be new (presented in that problem) or suitable for review. This proposal does not contain any such introductory material, and most proposals will not be accepted without some discussion demonstrating the proposer understands the problem. On the positive side, if the proposer understands the problem and includes a good discussion demonstrating that understanding, then there is a good chance the proposer can solve the problem. So we're not off to such a good start here, but let's see what we've got.

The first line is directly from the statement of the problem:

Assume $u : I \to \mathbb{R}$ is a monotone non-decreasing function on an interval I.

Appropriate introductory material to include would be the following:

1. Something about **intervals**. The preceding problem 10, for example classifies intervals as having one of ten possible forms. These will presumably constitute some kind of cases or be significant for this problem.

2. The definition of a **monotone function**.

3. The full statement of the problem. This problem goes on to say that the objective of the problem is to show the set of discontinuities of such a function is (at most) countable. This objective is not stated in the proposal.

4. In view of the full statement, one could/should also explain what is a **discontinuity** and what is the **set of discontinuities**.

5. Finally, one might mention what is involved in showing a set is **countable**.

None of these things are included in this proposal.

With a relatively difficult problem like this one (which follows four other preparatory problems), it's probably a good idea to explain the basic idea of your solution up front without details and then go back and fill in the details according to the strategy you've outlined. At least sometimes that can be a good idea. Ask yourself: What are the basic ideas? Otherwise, the reader is faced with content that comes, more or less, out of the blue.

Explicitly, we find:

Let us consider $I = \{n \in \mathbb{R} : a \leq n \leq b\}$
Let $c \in$

$$a, b$$

Then we can define a function:...

Okay, let's pause here and contemplate what has been submitted for our consideration above. I'm guessing there is some kind of problem with tex/latex manifest here starting in the second line.[1] Obviously, you should preview what you produce and make sure it expresses what you want to say in the manner you want to say it. You should read your proposal over carefully before you submit it. This second line does not indicate such care and concern for the reader.

There is no period at the end of the first line nor the second line. Have punctuation marks gone out of style?

We are not told the possible values of a and b. According to what I understand about intervals, the values of a and b, as indicated by the non-strict inequalities in the set description of the interval I, must be finite real numbers. This would be in harmony with my guess about the tex/latex issue. In any case, the interval I is evidently "considered" to be a finite closed interval. The problem with this consideration is that the interval I is given in the problem and is not given as a finite closed interval but simply an interval. Thus, to assume I is a finite closed interval is incorrect. But let's say c is in this interval, because that is what is probably intended, and see what comes next.

$$f(c) = \lim_{x \to c^+} u(x) - \lim_{x \to c^-} u(x)$$

The problems with this line are many. First it could be mentioned that a function is a rule or correspondence which assigns values to **arbitrary** elements of a set. In this case, we have a **fixed** element of a set c, so the only time this line could serve as the definition of a function would be if the function f had domain the singleton $\{c\}$. Perhaps that is what is intended.

Overlooking this first objection, as far as I know we have not defined limits of any sort. The statement of this problem does not contain any limits or any discussion of limits. The problems preceding it do not contain any limits or discussion of limits.

[1]Probably "\[" and "\]" (the symbols for a displayed equation) were used instead of "$ [a,b] $" for a closed interval which was what was intended.

The author does not provide any discussion of limits nor define limits. Thus, I think it is pretty clear that without any definition or discussion of the notion of limits, in this context, the use of limits is off limits. Of course, one could introduce the notion of limits. That might be okay, but it would be complicated and involved. I might suggest consideration of the previous problems instead.

The previous problems do give a simple, self-contained, discussion of concepts that would adequately handle what the author is trying to get at with limits here. Giving some attention to the context of the problem (based on the other problems) and what fits in with the discussion and flow of ideas in the course is probably a good idea.

Having considered this attempted definition, which is ended with no punctuation, we find the following:

Note that $f(c)$ is positive since u is non-decreasing (no period)
Thus, $f(c) \leq u(b) - u(a)$ (no period) Further...

The first assertion is possibly okay, though without definitions, we can't really get to it. With definitions, it would require a proof. We do not have a proof.

When one says "Thus,..." it is usually expected that the ensuing assertion follows from something previously stated. That does not seem to be the case here. My impression is that the word "thus" is being used in an anomalous sense, to mean something like, "I'm just going to start throwing out random assertions." In fact, it may be true that $f(c) \leq u(b) - u(a)$ (were a and b in the domain of u and f well-defined) but that assertion is, as far as I can see, pretty independent from the preceding assertion that $f(c)$ is positive.

It is at this point, where we encounter something rather unexpected.

...Further, the sum of all $f(c) \leq u(b) - u(a)$ (no period)

"**All** $f(c)$"??? The **sum** of all $f(c)$??? All one of them? Then why say "all?"

After contemplating this for a while, I'm pretty sure what is in mind here is

$$\sum_{c \in [a,b]} f(c)$$

where c is any point in an interval $[a, b]$ with $b - a > 0$ and f is a well-defined function measuring the jump in the function u at c. Then we're looking at a sum of uncountably many values. I would say it's pretty clear that such an exotic object requires at least some explanation. Nevertheless, this is an idea (a pretty good idea) which can be made sensible. Unfortunately again, even with the appropriate definitions, we would need some proof of the basic inequality

$$\sum_{c\in[a,b]} f(c) \le u(b) - u(a).$$

Moving to the last line after our last periodless pause, we find:

Then we can define the set

$$F(n) = \{\text{the set of all } f(c) \ge 1/n\} \text{ for } n \in \mathbb{N}$$
$$\text{and}$$
$$F = \{\text{the set of all } f(c)\}$$

...ending with no period, of course. This is not correct use of set notation. When we use the curly brackets "{ }," they are already declaring "the set of all." Thus, we have here "The set of all...the set of all..." which makes no sense. What the author means is to define the sets

$$F_n = \{c \in [a,b] : f(c) \ge 1/n\}$$

and

$$F = \cup_{n\in\mathbb{N}} F_n.$$

These are reasonable sets to consider. I think we have a problem or problems in Assignment 1B dedicated to following this approach. There, one shows the set F here (more or less) is the set of discontinuities. In that case, it is argued that there are only finitely many points in each F_n first, so that no sums of infinite (nor yet uncountable) collections of real numbers need be considered.

Then comes a proof of Theorem 1.4 from Gunning's textbook which states that *a countable union of countable sets is countable.* I'm not totally happy with the proof presented here (which I think is not so different from Gunning's), but I'm not not happy with Gunning's proof either. We could use a good proof of this result.

In any case, we can (and probably should) assume that result for this problem. Given that, we need (in summary)

1. F is the set of discontinuities.
2. Each F_n is countable.

The basic outline is workable, but some details and corrections are required.

3 Other Comments

This proposal had some good and workable ideas, but it needs a lot of work. A good deal of introductory material is needed. There are a lot of notational problems and some conceptual problems.

I hope the author, or some other author can help remove the red and replace it with blue.

4 Score out of 20

If I had to grade this problem on an exam, it might come in a little below 10 based on the ideas it contains, and the fact that they could be made into a viable solution. From another point of view, I might say: introduction: 0, readability: 0, correct mathematical expression: 0, ideas: 5. That would be a bit harsh, but you get the idea: Scoring would come in between 5 and 10 (probably not inclusive), and that's a score no one wants to get or give.