Continuity and Uniform Continuity

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Can you complete the following tasks?

1 Uniform Continuity

Starter 1 *Define what it means for a function* $f : \mathbb{R} \to \mathbb{R}$ *to be* **continuous at a point** $x_0 \in \mathbb{R}$.

Problem 1 *Show that the function* $f : \mathbb{R} \to \mathbb{R}$ *by* $f(x) = x^2$ *is continuous at* $x_0 = 5$.

Starter 2 *Let* $[0, 1] = \{x : 0 \le x \le 1\}$ *denote the closed unit interval in* $\mathbb{R}$ *and let* $(0, 1) = \{x \in \mathbb{R} : 0 < x < 1\}$ *denote the open unit interval in* $\mathbb{R}$.

1. *Define what it means for a function* $f : [0, 1] \to \mathbb{R}$ *to be* **continuous**.
2. *Define what it means for a function* $f : (0, 1) \to \mathbb{R}$ *to be* **uniformly continuous**.

Problem 2

1. *Give an example of a function* $f : (0, 1) \to \mathbb{R}$ *which is continuous but not uniformly continuous.*
2. *Prove that any function* $f : [0, 1] \to \mathbb{R}$ *which is continuous is uniformly continuous.*

Starter 3 *Define what it means for a function* $f : (0, 1) \to \mathbb{R}$ *to be* **bounded**.

Problem 3 *Give an example of a function* $f : (0, 1) \to \mathbb{R}$ *which is bounded and continuous but not uniformly continuous.*

Starter 4 *Define what it means for a subset of* $\mathbb{R}$ *to be* **compact**.

Problem 4 *Show that any function* $f : K \to \mathbb{R}$ *which is continuous on a compact subset* K *of* $\mathbb{R}$ *is uniformly continuous on* K.