MATH 4317
Administrative Details

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1 Objective(s) and Difficulty

This should be a difficult course for most of you. Part of the purpose of this course—maybe the main purpose—is to give you the opportunity to *learn how to think like a mathematician.* This is not an easy thing to learn for most people, but to **think** *precisely,* if not *critically,* is a skill that may offer some advantages. And this should probably be your main objective in the course, so be patient and give it a try. Closely related to this objective is the ability to communicate precisely or express your thinking in a *rigorous* manner. This is partially a question of language. There is going to be significant vocabulary, and you will need to think carefully about how you use words. In addition, you will be required to **write** your arguments carefully, precisely, and comprehensibly in English. You should think in terms of writing approximately twenty (short) term papers during the semester. Each of these may take several hours.

Beyond that, the opportunity to learn to think rigorously and to communicate your thoughts to others is presented within a certain set of mathematical results about sets, functions, regularity, convergence, uniformity, and various related structures and concepts. These are the topics of the subject of **analysis** or **advanced calculus** where rigorous mathematical thinking was first applied comprehensively and communicated by various mathematicians, especially Karl Weierstrass. So as a secondary objective, you will have the opportunity to become familiar with these topics from analysis. You can get some familiarity with these topics without learning how to think like a mathematician, but there is little point in doing that. You can learn how to think like a mathematician in the context of other subjects, but this subject of analysis is probably one of the best. Many mathematicians think it is the best.

If the paragraphs above do not describe your objectives for the course and you are not prepared to devote more attention to this course than any other mathematics course you have taken, you should drop the course now. If you do not plan to attend all the class meetings, maintain engagement both with the material and conversation, and participate, then you should drop the course now.

2 Structure

There are various approaches to teaching analysis. A famous one, and the one which my teacher used, was to give points for student presentation of proofs in class—with absolutely no input from a textbook, no lectures, and no help or hints. The proofs were proofs of assertions (lemmas, theorems, corollaries) from a list prepared by the instructor. The disadvantage was that progress was slow and few results were covered. On the other hand, the strategy was that if one can learn to think, then one can learn material easily later, so learning a lot of the material of analysis was not an important part of the course or a main objective. That strategy can work. It will not be the strategy used in this course.

What I suggest is the following: I will prepare 5 "assignments" (essentially lists of assertions). Each will have two parts (part A and part B) consisting of about 15 problems each. That means approximately $15 \times 5 \times 2 = 150$ problems for the semester or about 10 problems per week. You should try to **understand** solutions of all these problems (and more), but you may not be able to work on each problem on your own "from scratch." There are too many of them. But you should try to tackle at least four or five of them on your own "from scratch" each week. For as many of these problems as you can, you should prepare a written solution (preferably typed in latex in a 12 point article format). This will serve as a proposal for making a presentation in class. I, as the instructor, will either approve or reject your solution. Note: An approval of a solution does **not** mean the solution is correct. It simply means you are approved to make a presentation in class, and I will record a time on the schedule page for you to make your presentation. Seeing (and even making/presenting) incorrect solutions should be an important part of the learning process for this course. There is nothing wrong, per se, with giving a proof which has some flaw or error—though of course, this should not be your objective. But if that happens, it's nothing to really worry about.

Your in-class presentations will count for your "class participation" and will account for 20% of your course grade (though you shouldn't really worry about grades—grades will not be the point of this course). If you think getting a grade is the point of the course, then you should drop the course now. The important thing is that you should make in-class presentations and you should participate in the discussion of the presentations of others. Example solution proposals may be found on the course page(s).

Important: The in-class presentations are **not** restricted to the problems listed on the assignments. You may use any problems from the text, from notes I provide, or from other topics you find interesting (related to the course material) from anywhere. Any of these may be the source for a proposal.

If approved, you should plan on presenting your problem/solution/proof in about 10 to 20 minutes in class. (The time may vary, and might be more or less than these typical times.) Both your proposal and your presentation should include an introduction to the problem so the problem is clear to those who read or listen (and it is clear to the reader or listener that the problem is clear to you).

After about three weeks, I will compile a selection of the problems from the assignment, and these will constitute a take-home exam. You should write up solutions for all these problems. Generally speaking, if there has been no "successful" presentation of a problem in class, i.e., if the problem remains "open," then that problem will be on the exam. Problems which have been thoroughly discussed and solved in class will not be on the exam. If all problems have been thoroughly discussed and solved in class, then *there will be no exam.*

Occasionally I have a student who is shy about standing up to make presentations in class because he feels his English skills are not adequate or for some other reason. You should not continue in this course this semester if you feel this way, and especially if you are not planning to participate by making presentations and then tell me about it at the end of the semester. If you don't want to make presentations in class, that is fine. But you need to drop the course now and take it some other semester with some other instructor. There are other instructors who teach this course, and I'm sure you can find one to your liking.

3 Order

In principle, it is very important for you (and me) to keep track of the order in which results are proved and the order in which various results depend on one another. If one is not careful, then one can fall victim to "circular reasoning" or proving something based on the assumption that it is true. For various reasons, however, it is not convenient to always think about things (or present results and ideas) in a strict order. One of these reasons is that, in certain respects, we are jumping into the middle of mathematics and we are not always going to chase the "foundations" of our work to the "simplest" possible starting point. Underlying the subject of analysis are the subjects of serious set theory and logic, and somewhere underlying those is darkness. That is to say, when one attempts to find "foundations" for analysis the pursuit does not get simpler and, at length, what people have been able to find—if one can understand it—is not altogether satisfying. Fortunately, for the most part we can explain fairly precisely and carefully certain aspects of logic, set theory, and starting structures (like numbers, assumptions of existence, and notions of induction) which are rather compelling and natural for most people. This is what I plan to do.

In addition to this "big skip" in foundations (which is sort of inevitable) we will skip around a little bit simply to make the course more interesting and to cover more material. You should always, therefore, keep in mind the organization and order of what we are doing *as much as you can,* with the realization that you may not understand everything in a chronologically strictly "linear" fashion; some understanding may come to you retrospectively. And certainly your objective should be to piece together a clear sequential dependence through which all your mathematical reasoning can be traced.

4 Reading

You should try to read the textbook. This will also not be too easy. (And if you find it easy, then it is very likely you are not reading it carefully enough and you are not understanding what it says very much at all.) This is not because the book is defective. The book is pretty good. It is because reading mathematics requires careful attention and hard thinking (at a level to which you are probably not used). And even when you get used to it, it can still be hard going, even for people (and mathematicians) who know the subject well.

Summary

Hopefully these comments, more or less, cover all the administrative details for the course you need to know. If you want to learn analysis, and you try to learn analysis, then I think you will be successful. If you have any other questions, just ask me.