Challenge question: Draw nice accurate digital image of these planes intersecting

a)
$$\begin{vmatrix} x + 2y + 3z = 1 \\ x + 3y + 4z = 3 \\ x + 4y + 5z = 4 \end{vmatrix}$$

The plot of the 3 planes intersecting is shown in the following figure. As seen from the plot, lines of intersection occurs between 2 planes at most. Since there are 3 unknowns, there is no solution that exists where 3 equations are satisfied simultaneously (intersection of 3 planes). Hence this system of equations has no solution.

Graphical representation of planes
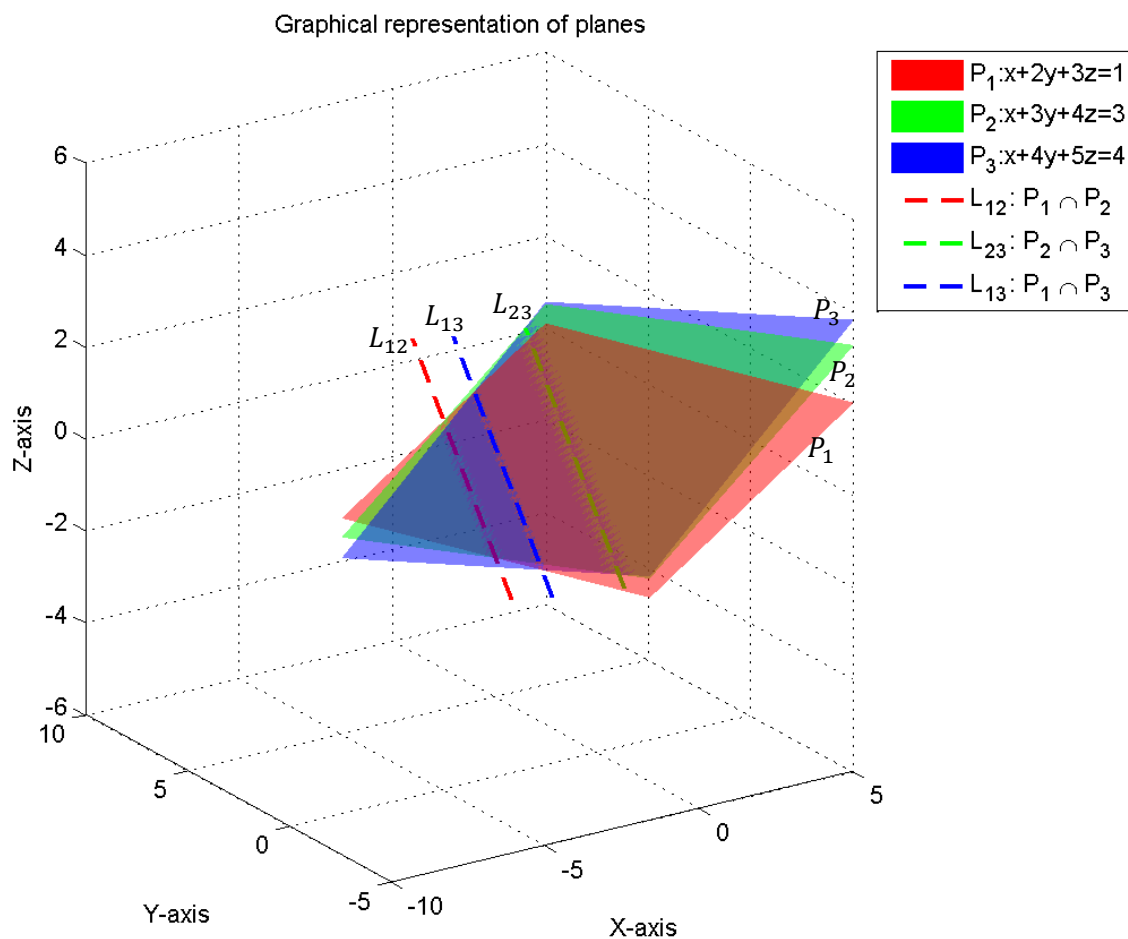
b) $\begin{vmatrix} x + 2y + 3z = 1 \\ 3x + 2y + z = 1 \\ 7x + 2y - 3z = 1 \end{vmatrix}$

The plot for the above system of equations is given below, with 1 line of intersection between 3 planes. Hence, this system is consistent and has infinite number of solutions lying along L_{123} with equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

Graphical representation of planes

P_1:x+2y+3z=1
P_2:3x+2y+z=1
P_3:7x+2y-3z=1
L_{123}: $P_1 \cap P_2 \cap P_3$