Linear Algebra Challenge

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The objective is to accurately illustrate the intersection of the three planes

$$x + 2y + 3z = 1,$$
$$3x + 2y + z = 1, \text{ and}$$
$$7x + 2y - 3z = 1.$$

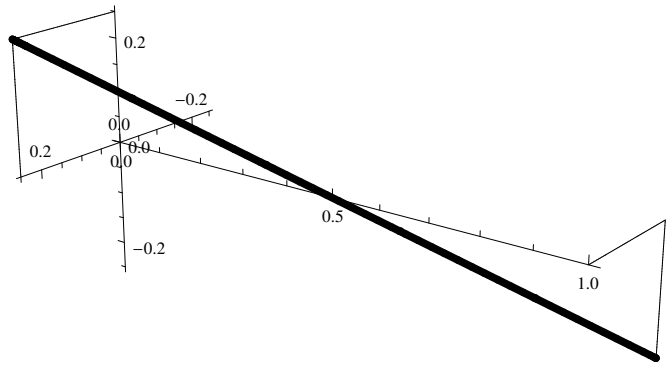
It was shown in class that the intersection is a straight line parameterized by

$$x = t$$
$$y = 1/2 - 2t$$
$$z = t$$

We will illustrate the solution of the system with several graphics. The first is a plot of the intersection line:

In[1]:=
```
Show[ParametricPlot3D[{t, 1 / 2 - 2 t, t}, {t, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0},
  ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2}, Boxed → False, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 1 / 4}, {1 / 4, 0, t}, {-t, 1, 0}, {-1 / 4, 1, -t}},
  {t, 0, 1 / 4}, PlotStyle → Dashed]]
```

Out[1]=



One can see here the intersection of the line with the y axis at (0,1/2,0) and the intersection with the x,z-plane at (1/4,0,1/4).

In order to get a balanced view of the planes, we will use a vector in each plane perpendicular to the intersection line. These vectors can be obtaind by the cross product.

We start with the vector in the direction of the intersection line:

In[2]:=
```
v = {1, -2, 1};
```

Then we take the normals to each plane:

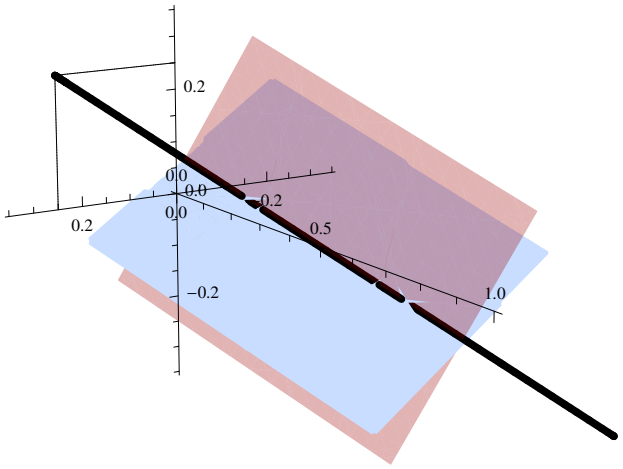
In[3]:=
```
n1 = {1, 2, 3};
n2 = {3, 2, 1};
n3 = {7, 2, -3};
```

In[6]:=
```
w1 = Cross[v, n1] / Norm[Cross[v, n1]];
w2 = Cross[v, n2] / Norm[Cross[v, n2]];
w3 = Cross[v, n3] / Norm[Cross[v, n3]];
```

Now we show the intersection line with portions of the first two intersecting planes. To distinguish the two planes, we will make the first one opaque and the second one see through (and red).

In[9]:=
```
Show[ParametricPlot3D[{t, 1 / 2 - 2 t, t}, {t, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0},
  ViewPoint → {4.4 / 2, 3.4, 1}, Boxed → False, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 1 / 4}, {1 / 4, 0, t}}, {t, 0, 1 / 4}, PlotStyle → Dashed],
 ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w1, {t, -1 / 8, 1 / 8}, {s, -1 / 4, 1 / 4} ,
  AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2}, Boxed → False,
  Mesh → False], ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w2, {t, -1 / 8, 1 / 8},
  {s, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2},
  Boxed → False, Mesh → False, PlotStyle → {Red, Opacity[.3]}], PlotRange → All]
```

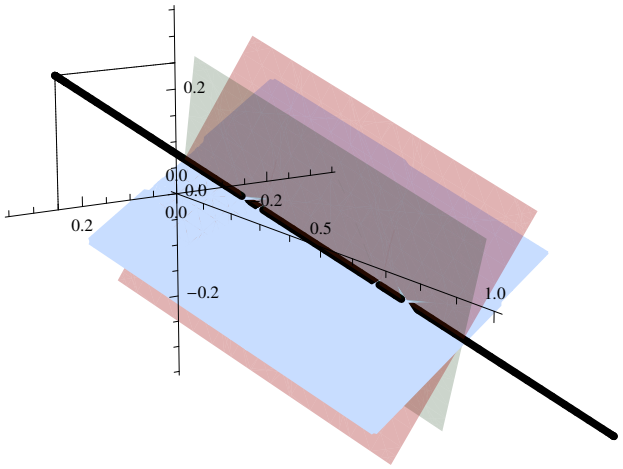
Out[9]=



Finally, we add the third plane:

In[10]:=
```
Show[ParametricPlot3D[{t, 1 / 2 - 2 t, t}, {t, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0},
  ViewPoint → {4.4 / 2, 3.4, 1}, Boxed → False, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 1 / 4}, {1 / 4, 0, t}}, {t, 0, 1 / 4}, PlotStyle → Dashed],
 ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w1, {t, -1 / 8, 1 / 8}, {s, -1 / 4, 1 / 4} ,
  AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2}, Boxed → False,
  Mesh → False], ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w2, {t, -1 / 8, 1 / 8},
  {s, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2},
  Boxed → False, Mesh → False, PlotStyle → {Red, Opacity[.3]}],
 ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w3, {t, -1 / 8, 1 / 8}, {s, -1 / 4, 1 / 4} ,
  AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2}, Boxed → False,
  Mesh → False, PlotStyle → {Green, Opacity[.2]}], PlotRange → All]
```

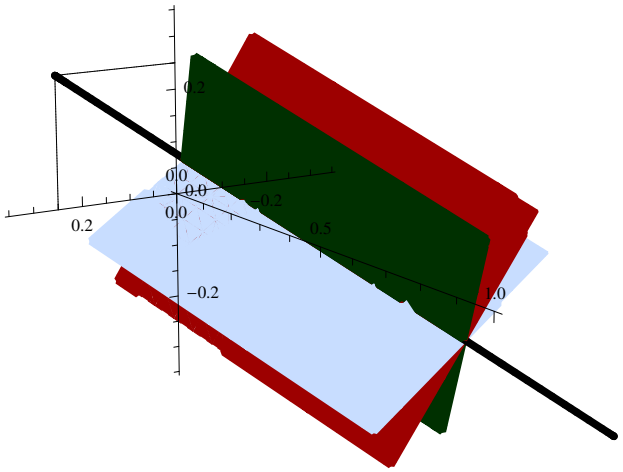
Out[10]=



And with all three planes opaque:

In[11]:=
```
Show[ParametricPlot3D[{t, 1 / 2 - 2 t, t}, {t, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0},
  ViewPoint → {4.4 / 2, 3.4, 1}, Boxed → False, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 1 / 4}, {1 / 4, 0, t}}, {t, 0, 1 / 4}, PlotStyle → Dashed],
 ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w1, {t, -1 / 8, 1 / 8}, {s, -1 / 4, 1 / 4} ,
  AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2}, Boxed → False,
  Mesh → False], ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w2, {t, -1 / 8, 1 / 8},
  {s, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2},
  Boxed → False, Mesh → False, PlotStyle → {Red, Opacity[1]}],
 ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w3, {t, -1 / 8, 1 / 8}, {s, -1 / 4, 1 / 4} ,
  AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2}, Boxed → False,
  Mesh → False, PlotStyle → {Green, Opacity[1]}], PlotRange → All]
```

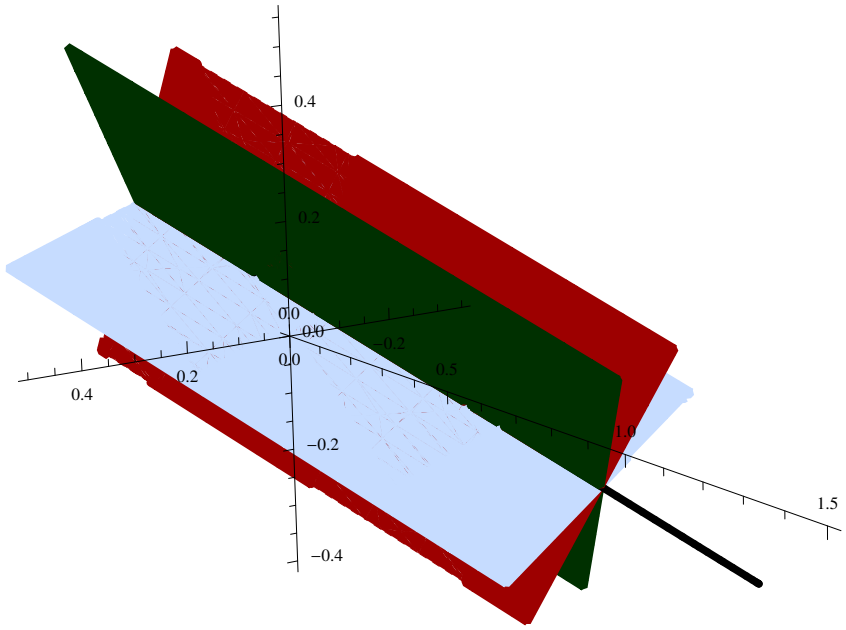
Out[11]=



One thing that looks wrong is that the green/black plane looks like it is tilting back away from the viewer, but in fact, if we restrict to y = 0, we get 7x -3 z = 1 which is a line in the x,z-plane with positive slope. We can illustrate that the third plane is actually tilting toward the viewer by extending the planes back to meet y = 0:

In[12]:=
```
Show[ParametricPlot3D[{t, 1 / 2 - 2 t, t}, {t, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0},
  ViewPoint → {4.4 / 2, 3.4, 1}, Boxed → False, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 1 / 4}, {1 / 4, 0, t}}, {t, 0, 1 / 4}, PlotStyle → Dashed],
 ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w1, {t, -1 / 8, 2 / 4}, {s, -1 / 4, 1 / 4} ,
  AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2}, Boxed → False,
  Mesh → False], ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w2, {t, -1 / 8, 2 / 4},
  {s, -1 / 4, 1 / 4} , AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2},
  Boxed → False, Mesh → False, PlotStyle → {Red, Opacity[1]}],
 ParametricPlot3D[{t, 1 / 2 - 2 t, t} + s w3, {t, -1 / 8, 2 / 4}, {s, -1 / 4, 1 / 4} ,
  AxesOrigin → {0, 0, 0}, ViewPoint → {4.4 / 2, 3.4 / 2, 2 / 2},
  Boxed → False, Mesh → False, PlotStyle → {Green, Opacity[1]}],
 PlotRange → {{-0.35, 0.5}, {0, 1.5}, {-0.4, 0.55}}]
```

Out[12]=



The second intersection involves three planes with no common intersection (no solution):

x + 2y + 3z = 1
x + 3y + 4z = 3
x + 4y + 5z = 4.

In this case, the three planes intersect pairwise in three parallel lines. We can find the parallel lines by intersecting the planes pairwise:

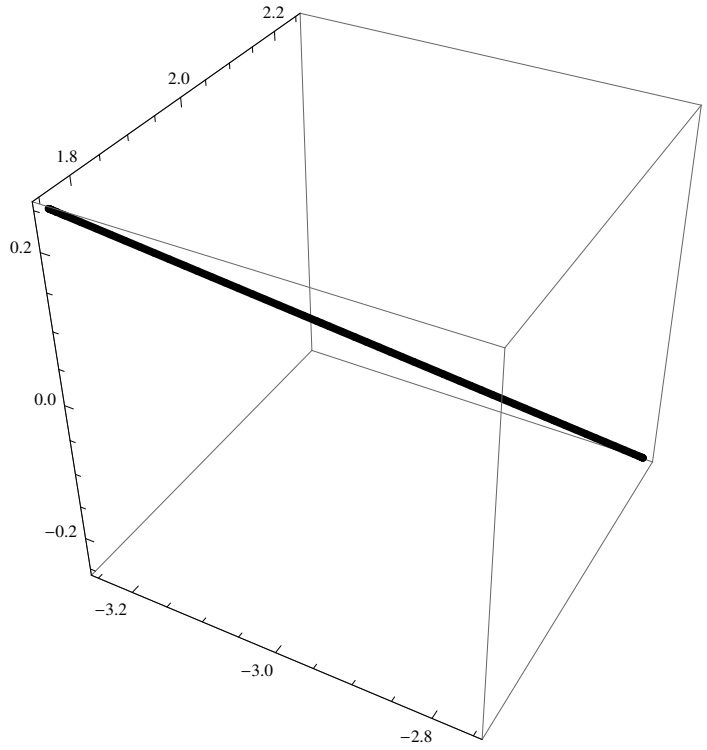
x + 2y + 3z = 1
x + 3y + 4z = 3

has intersection

x = -3 - t
y = 2 - t
z = t

In[13]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t},
  {t, -1 / 4, 1 / 4}, PlotStyle → AbsoluteThickness[4]] ]
```

Out[13]=



The second and third equations

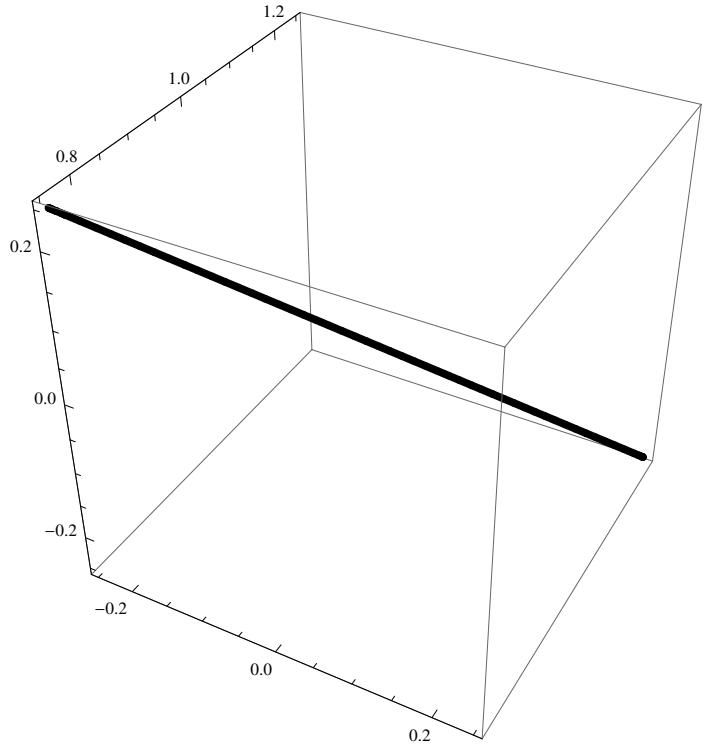
x + 3 y + 4 z = 3
x + 4 y + 5 z = 4

have intersection

x = - t
y = 1 - t
z = t

In[14]:=
```
Show[ParametricPlot3D[{-t, 1 - t, t},
  {t, -1 / 4, 1 / 4}, PlotStyle → AbsoluteThickness[4]] ]
```

Out[14]=



The first and third equations

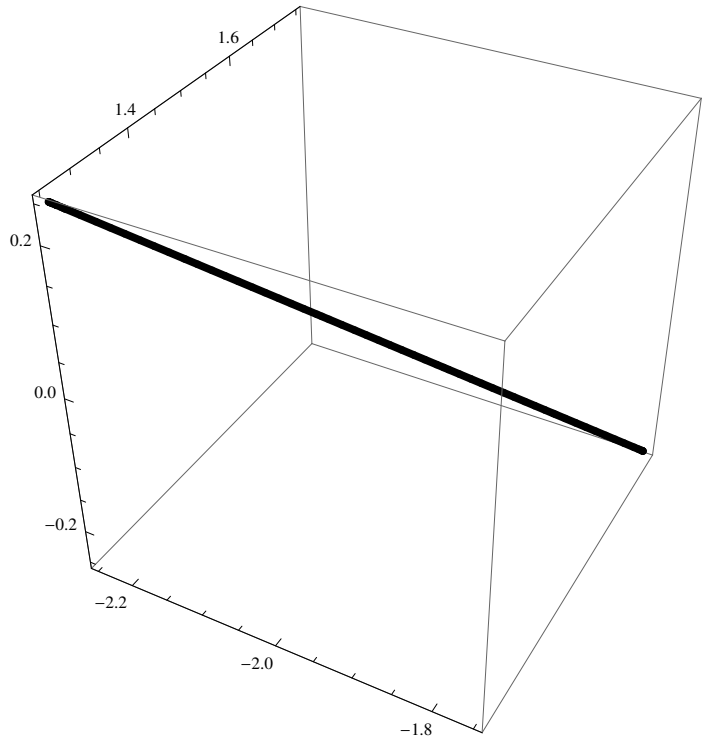
x + 2 y + 3 z = 1
x + 4 y + 5 z = 4

have intersection

x =-2 - t
y = 3/2 - t
z = t

In[21]:=
```
Show[ParametricPlot3D[{-2 - t, 3 / 2 - t, t},
  {t, -1 / 4, 1 / 4}, PlotStyle → AbsoluteThickness[4]] ]
```

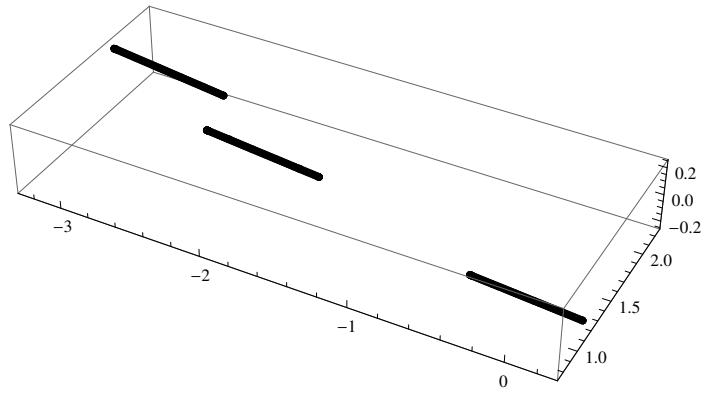
Out[21]=



Plotting these three lines together we have

In[22]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 4, 1 / 4},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-t, 1 - t, t}, {t, -1 / 4, 1 / 4}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{-2 - t, 3 / 2 - t, t}, {t, -1 / 4, 1 / 4},
  PlotStyle → AbsoluteThickness[4]], PlotRange → All]
```

Out[22]=



Obviously, it is going to take some work to illustrate this intersection.

Each line is in the direction {-1, -1, 1}. The plane through the origin orthogonal to this direction is

-x - y + z = 0 or z = x + y.

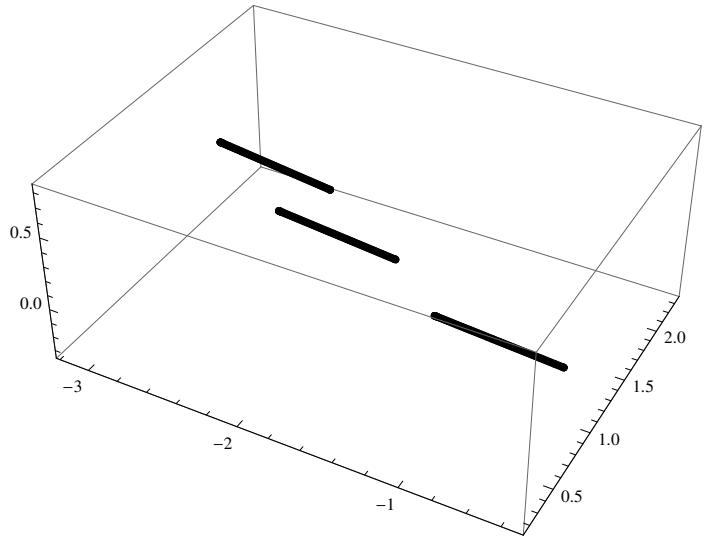
The intersections of the lines with this plane are

{-8/3, 7/3, -1/3} corresponding to t = -1/3,
{-1/3, 2/3, 1/3} corresponding to t= 1/3, and
{-11/6, 5/3, -1/6} corresponding to t = -1/6

First we will plot segments from these same lines starting from these co-planar points.

In[37]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + 0.5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5},
  PlotStyle → AbsoluteThickness[4]], PlotRange → All]
```

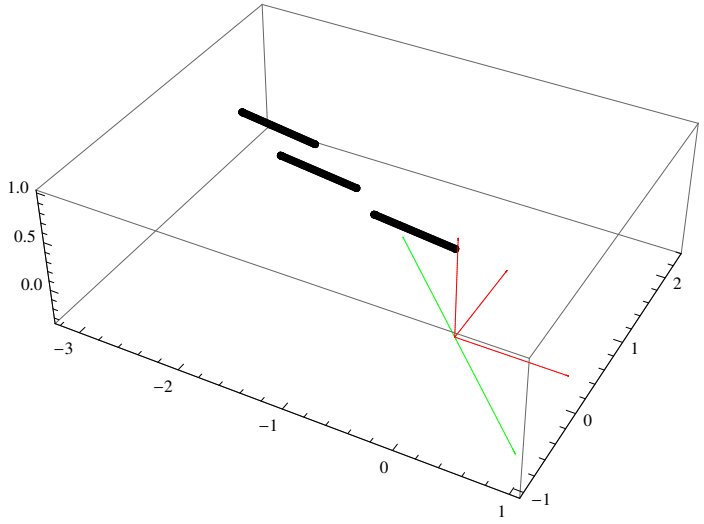
Out[37]=



This looks sort of OK. But it's not too clear where these lines are located with respect to the coordinate axes and what is the viewpoint.

In[38]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0}, {t, -1, 1}, PlotStyle → Green], PlotRange → All]
```

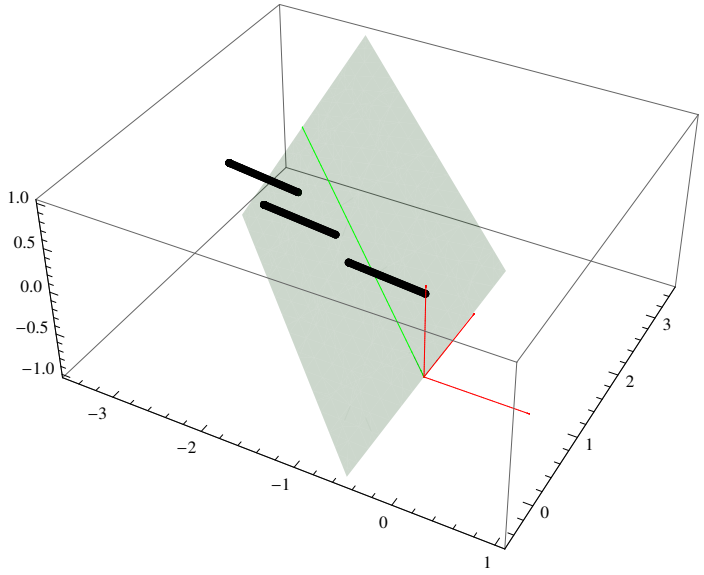
Out[38]=



It's still not very clear. We will try to put in the plane z = x + y to help see the location of these segments.

In[45]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + 0.5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.2]}, Mesh → False],
 ParametricPlot3D[{t, -t, 0}, {t, -3, 0}, PlotStyle → Green], PlotRange → All]
```

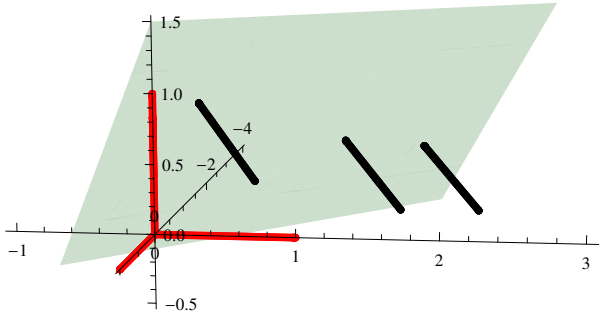
Out[45]=



One can see the position pretty well now. The segments start on the plane and extend toward the viewer. Let's try to rotate the viewpoint and move the orthogonal plane to the other ends of the segments.

In[69]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2} + 0.5 {-1, -1, 1}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.2]}, Mesh → False],
 ParametricPlot3D[{t, 0, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, t, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, 0, t}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 PlotRange → All, ViewPoint → {2, .2, 0.3}, Boxed → False, AxesOrigin → {0, 0, 0}]
```

Out[69]=



Finally, we will use the method before to indicate the intersecting planes:

In[70]:=
```
vnew = {-1, -1, 1};
```

Then we take the normals to each plane:

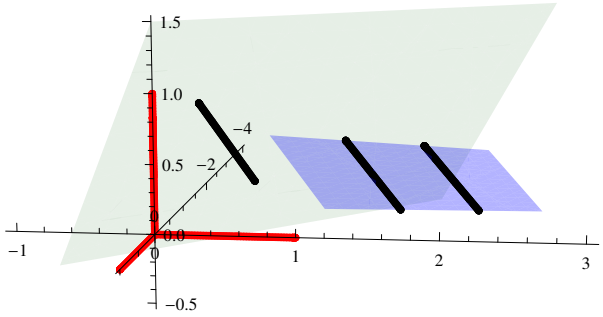
```
m1 = {1, 2, 3};
m2 = {1, 3, 4};
m3 = {1, 4, 5};
```

In[74]:=
```
w1new = Cross[vnew, m1] / Norm[Cross[vnew, m1]];
w2new = Cross[vnew, m2] / Norm[Cross[vnew, m2]];
w3new = Cross[vnew, m3] / Norm[Cross[vnew, m3]];
```

Here is the firrst plane.

In[87]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2} + 0.5 {-1, -1, 1}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.1]}, Mesh → False],
 ParametricPlot3D[{t, 0, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, t, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, 0, t}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{-3 - t, 2 - t, t} + s w1new, {t, -1 / 3, -1 / 3 + 0.5},
  {s, -2, 1} , Mesh → False, PlotStyle → {Blue, Opacity[.3]}], PlotRange → All,
 ViewPoint → {2, .2, 0.3}, Boxed → False, AxesOrigin → {0, 0, 0}]
```

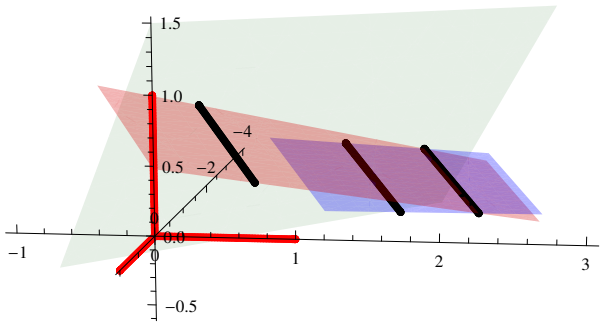
Out[87]=



Now the firrst and second planes.

In[94]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2} + 0.5 {-1, -1, 1}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.1]}, Mesh → False],
 ParametricPlot3D[{t, 0, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, t, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, 0, t}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{-3 - t, 2 - t, t} + s w1new, {t, -1 / 3, -1 / 3 + 0.5},
  {s, -2, 1} , Mesh → False, PlotStyle → {Blue, Opacity[.3]}],
 ParametricPlot3D[{-t, 1 - t, t} + s w2new, {t, 1 / 3, 1 / 3 + 0.5}, {s, -1, 4} ,
  Mesh → False, PlotStyle → {Red, Opacity[.3]}], PlotRange → All,
 ViewPoint → {2, .2, 0.3}, Boxed → False, AxesOrigin → {0, 0, 0}]
```

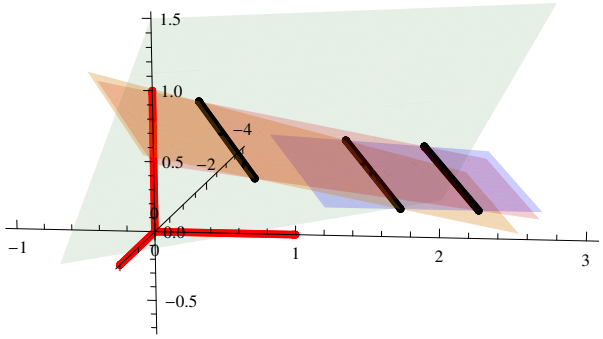
Out[94]=



And all three planes.

In[101]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2} + 0.5 {-1, -1, 1}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.1]}, Mesh → False],
 ParametricPlot3D[{t, 0, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, t, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, 0, t}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{-3 - t, 2 - t, t} + s w1new, {t, -1 / 3, -1 / 3 + 0.5},
  {s, -2, 1} , Mesh → False, PlotStyle → {Blue, Opacity[.2]}],
 ParametricPlot3D[{-t, 1 - t, t} + s w2new, {t, 1 / 3, 1 / 3 + 0.5},
  {s, -1, 4} , Mesh → False, PlotStyle → {Red, Opacity[.2]}],
 ParametricPlot3D[{-2 - t, 3 / 2 - t, t} + s w3new, {t, -1 / 6, -1 / 6 + 0.5},
  {s, -3, 2} , Mesh → False, PlotStyle → {Orange, Opacity[.3]}],
 PlotRange → All, ViewPoint → {2, .2, 0.3}, Boxed → False, AxesOrigin → {0, 0, 0}]
```

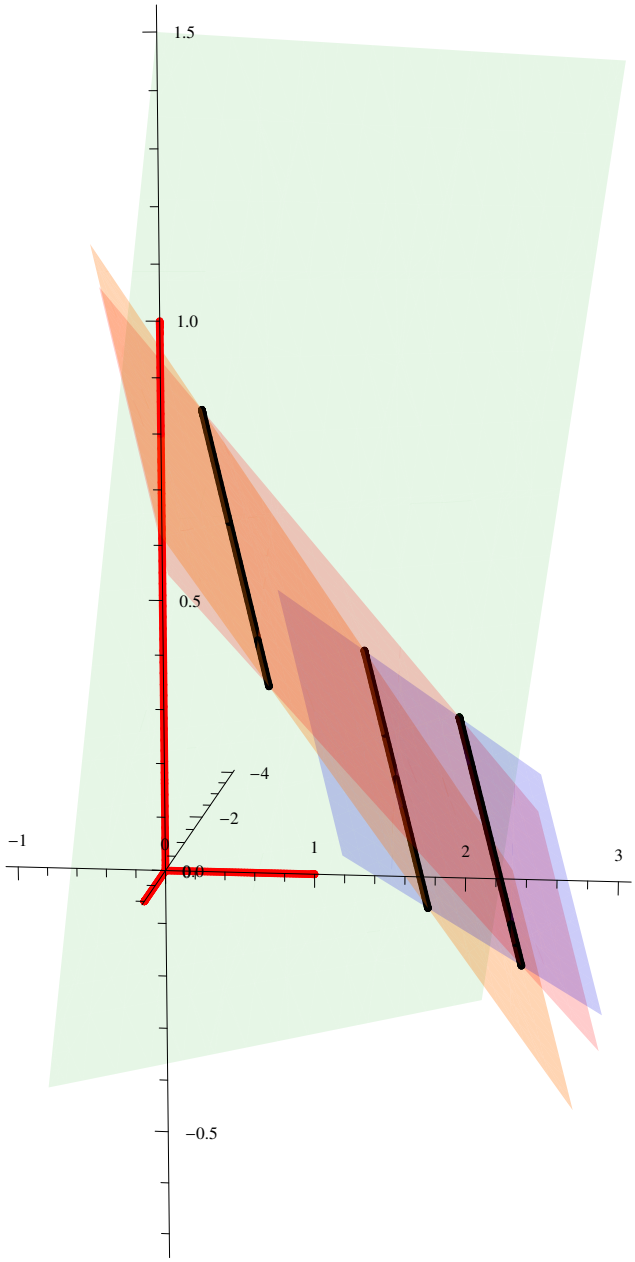
Out[101]=



Since the triangular region enclosed by the three planes is somewhat “thin,” we can also scale up the z direction (by 2) to try to see it better.

In[111]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2} + 0.5 {-1, -1, 1}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.1]}, Mesh → False],
 ParametricPlot3D[{t, 0, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, t, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, 0, t}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{-3 - t, 2 - t, t} + s w1new, {t, -1 / 3, -1 / 3 + 0.5},
  {s, -2, 1} , Mesh → False, PlotStyle → {Blue, Opacity[.2]}],
 ParametricPlot3D[{-t, 1 - t, t} + s w2new, {t, 1 / 3, 1 / 3 + 0.5},
  {s, -1, 4} , Mesh → False, PlotStyle → {Red, Opacity[.2]}],
 ParametricPlot3D[{-2 - t, 3 / 2 - t, t} + s w3new, {t, -1 / 6, -1 / 6 + 0.5},
  {s, -3, 2} , Mesh → False, PlotStyle → {Orange, Opacity[.3]}], PlotRange → All,
 ViewPoint → {2, .2, 0.3}, Boxed → False, AxesOrigin → {0, 0, 0}, BoxRatios → {1, 1, 2}]
```

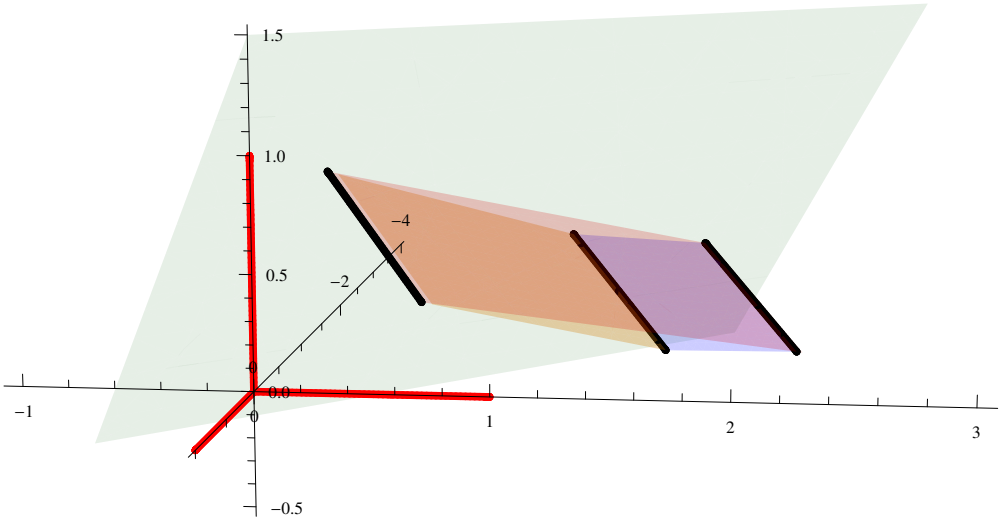
Out[111]=



Finally, we can trim away the extensions of the planes to see the triangular region better.

In[112]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2} + 0.5 {-1, -1, 1}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.1]}, Mesh → False],
 ParametricPlot3D[{t, 0, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, t, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, 0, t}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{-3 - t, 2 - t, t} + s w1new, {t, -1 / 3, -1 / 3 + 0.5},
  {s, -1.1, 0} , Mesh → False, PlotStyle → {Blue, Opacity[.2]}],
 ParametricPlot3D[{-t, 1 - t, t} + s w2new, {t, 1 / 3, 1 / 3 + 0.5},
  {s, 0, 3} , Mesh → False, PlotStyle → {Red, Opacity[.2]}],
 ParametricPlot3D[{-2 - t, 3 / 2 - t, t} + s w3new, {t, -1 / 6, -1 / 6 + 0.5},
  {s, -1.8, 0} , Mesh → False, PlotStyle → {Orange, Opacity[.3]}],
 PlotRange → All, ViewPoint → {2, .2, 0.3}, Boxed → False, AxesOrigin → {0, 0, 0}]
```

Out[112]=



In[109]:=
```
Show[ParametricPlot3D[{-3 - t, 2 - t, t}, {t, -1 / 3, -1 / 3 + .5},
  PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[{-t, 1 - t, t},
  {t, 1 / 3, 1 / 3 + 0.5}, PlotStyle → AbsoluteThickness[4]], ParametricPlot3D[
  {-2 - t, 3 / 2 - t, t}, {t, -1 / 6, -1 / 6 + 0.5}, PlotStyle → AbsoluteThickness[4]],
 ParametricPlot3D[{{t, 0, 0}, {0, t, 0}, {0, 0, t}}, {t, 0, 1}, PlotStyle → Red],
 ParametricPlot3D[{t, -t, 0} + s {1, 1, 2} + 0.5 {-1, -1, 1}, {t, -3, 0},
  {s, -0.5, 0.5}, PlotStyle → {Green, Opacity[0.1]}, Mesh → False],
 ParametricPlot3D[{t, 0, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, t, 0}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{0, 0, t}, {t, 0, 1}, PlotStyle → {Red, AbsoluteThickness[4]}],
 ParametricPlot3D[{-3 - t, 2 - t, t} + s w1new, {t, -1 / 3, -1 / 3 + 0.5},
  {s, -1.1, 0} , Mesh → False, PlotStyle → {Blue, Opacity[.2]}],
 ParametricPlot3D[{-t, 1 - t, t} + s w2new, {t, 1 / 3, 1 / 3 + 0.5},
  {s, 0, 3} , Mesh → False, PlotStyle → {Red, Opacity[.2]}],
 ParametricPlot3D[{-2 - t, 3 / 2 - t, t} + s w3new, {t, -1 / 6, -1 / 6 + 0.5},
  {s, -1.8, 0} , Mesh → False, PlotStyle → {Orange, Opacity[.3]}], PlotRange → All,
 ViewPoint → {2, .2, 0.3}, Boxed → False, AxesOrigin → {0, 0, 0}, BoxRatios → {1, 1, 2}]
```

Out[109]=