Project suggestion
Probability Mass Densities

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Consider the class of absolutely continuous probability mass densities δ with statistical range the interval $[0,1]$. Each such density corresponds to an integral probability measure $\pi : \mathfrak{M} \to [0,1]$.

There is a uniform measure that falls into this class, but aside from that, there do not appear to be any common, important, or canonical examples.

Can you give a "natural" example (or examples) of such a density with measures having prescribed mean and variance?

Are there universal bounds for the variance of the measure π? What if the additional assumption that the density itself is continuous is imposed? What if the density is continuous and is required to have at most one local maximum value or is in some other way constrained to be "bell shaped?"

Can such measures be obtained as limits of functions associated with measures on sets with finitely many elements like the induced binomial measures?