

# Poker Project(s)

John McCuan

October 4, 2023

Two students have asked me about pursuing projects related to poker card games. One is interested specifically in some kind of relationship between probability, strategy, and betting. There seems to be some subjective element with regard to the strategy. I do not understand this element and/or a specific strategy, so it is difficult for me to determine if there is a viable project in this direction or not.

The other student asked initially about calculating numbers of specific hands and associated probabilities. The results of some such calculations are summarized in a large table on the Wikipedia page

[https://en.wikipedia.org/wiki/Poker\\_probability](https://en.wikipedia.org/wiki/Poker_probability).

Some of the results in this table are discussed and checked in my notes and in homework Assignment 3. Certainly studying this table and verifying all the asserted numbers of hands and probabilities could be a reasonable topic for a project. This is not a very difficult project, and it could be improved by pushing a bit beyond the content in the table. On the other hand, really doing an easy project well is better than doing no project at all, and also doing an easy project well is probably better than just being confused about a vague and potentially difficult project. Working on a very difficult project, as long as the objective and topic of study are well defined and explained is, of course, fine, even if not much progress is made.

In any case, I think I have a suggestion which pushes the study of the table a little further and makes that direction into a rather better project. The first part of this project would be to study the table and verify the values in it. Once that is done, one can attempt to take each hand and attempt to calculate, subject to knowing that hand, **the probability that another hand in a particular game beats the given hand**. For example, say one has a straight flush consisting of 9, 10, *J*, *Q*, and *K* in spades. If you were looking at such a hand, you could ask: How many

hands are possible in a game that beat this hand? And what is the probability that another player has one of those hands? In this particular case, I believe the correct answers are the following: There are  $52 - 5 = 47$  cards left in the deck, and precisely 3 possible hands that beat the given straight flush in spades. Those are the three royal flushes in the remaining three suits (clubs, diamonds, and hearts). Since the total number of hands constructible from the remaining 47 cards is  $C(47, 5) = 1\,533\,939$ , the probability one of these hands is held by one other player is

$$\frac{1}{511\,313} = \frac{1}{11(23)(43)(47)}.$$

The probability that some other player in the game has such a winning hand depends on the number of other players. If there are 3 players total, then this probability should be doubled:

$$\frac{2}{511\,313} = \frac{1}{11(23)(43)(47)}.$$

And if there are  $k$  players, then the probability that the 9- $K$  straight flush in spades will lose is

$$\frac{k-1}{511\,313}.$$

Obviously, for a reasonable number of players, the probability of winning with this hand is quite high.

At some point, as one proceeds to hands of lesser value, the probability of being beaten increases and becomes greater than  $1/2$ . It would be especially interesting to know which hand or hands correspond to the transition from greater than  $1/2$  to less than  $1/2$ . Also, it would be interesting to know if the transition depends on the number of players  $k$  and, if so, to understand the dependence.

I think this is a relatively difficult but probably possible project. Most importantly, I think the question is well-defined and clear.

Another question, one nominally of strategy, comes to mind: Say you have  $n$  dollars to bet on such a game (and say you can play many many games of this sort). Knowing the probabilities of winning, can you devise a strategy for determining how much you should bet to maximize your winnings? This seems to be much more difficult, and I'm not immediately sure how to do it. The question may or may not be completely well-defined. I'm not sure about that either. But I am pretty sure that if one were to think carefully about this question the answers at least to the preliminary questions about whether the topic is well-defined and suitable for a project could be answered.