Project Example: MIT Problem 1 Probability of Poker Hands

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This is a problem set to go along with my discussion of MIT Problem 1 of Orloff and Booth which I have written up in a Mathematica notebook. These two documents constitute a good example of the documents suitable to go along with a project in probability for the course MATH 3215 in the Fall semester of 2023. If I can put these together with a twenty minute presentation, I'll have finished my project in probability. This one is probably of medium difficulty. Probably producing a large amount of clarity is better than taking on a large amount of difficulty.¹

Problem 1 In the card game of bridge a hand has thirteen cards.

- (a) Without making a calculation, determine if there are more bridge hands or more poker hands.
- (b) Calculate the number of (possible) bridge hands.
- (c) Bridge is played by four players designated "north," "south," "east," and "west." Let us say a "setup" is a pairing of four distinct bridge hands with the four players (i.e., directions). Assuming no two of the hands can have any single card in common, how many "setups" are there?

 $^{1}\mathrm{YMMV}$

Problem 2 Say 20 students are in room B9 in the Boggs building.

- (a) If there are five chairs in a row at the front of the room, how many different arrangements of five students can sit in the chairs.
- (b) A group or team of 5 students is selected to go outside the room and get five more chairs. How many different teams of five students can be formed to go get more chairs?
- (c) With ten chairs in a circle at the front of the room how many distinct orderings of ten students can sit in the circle if the particular chair in which a student sits does not matter but only the students sitting next to each student determine the arrangement.

Problem 3 The first row of Pascal's triangle is

$$\left(\begin{array}{c}0\\0\end{array}\right) = 1.$$

The second row of Pascal's triangle is

$$\left(\begin{array}{c}1\\0\end{array}\right) = 1 \qquad \left(\begin{array}{c}1\\1\end{array}\right) = 1.$$

What is the relation between the fifty-third row of Pascal's triangle and hands of cards? (See Problem 1 part (a) above.)

Problem 4 Calculate the number of poker hands classified as

- (a) four of a kind.
- (b) full house.

Problem 5 Calculate the number of poker hands classified as

- (a) straight not straight flush.
- (b) flush not straight flush.