Surge and Purge (Kaashyap & Erol[1], 2023)

Standard Game:

Teams A and B start with populations α and β respectively, $\alpha,\ \beta\ \in\ \mathbb{N}^{+}$. They play repeated matches against each other until one or both of their populations are zero.

During each match, the probability a team wins is proportional to its current (not initial) population. That is, Team A wins with probability $\frac{\alpha}{\alpha+\beta}$ and Team B wins with probability $\frac{\beta}{\alpha+\beta}$. After each match, two-thirds of the winning team dies, and one-half of the losing team dies. For example, if Team A won, set updated populations α to floor $(\frac{1}{3}\alpha)$ and β to floor $(\frac{1}{2}\beta)$.

When either team's population becomes 0, so $a\ =\ 0$ or $b\ =\ 0$, the game terminates. At this point, any teams with a positive population are victors, and any teams with zero population are losers. It is possible for more than one team to lose.

Generalizations:

a) Proportion of deaths: Instead of ⅓ and ½, the proportion of deaths is given by x_{win} and x_{loss} respectively, where $x_{win}, x_{loss} \in\ (0,\ 1)$. Another possible extension is to grant each team unique population rescaling coefficients, rather than having them fixed.
b) The game is played with N teams, with initial populations $p\ =\ (p_1, p_2, ..., p_n)$, where for each match, 1 team wins and $N\ -\ 1$ teams lose. The game end condition is when any team has zero population, which may be further generalized to when $n\ <\ N$ teams have zero population.

Problems:

a) Determine the probability that each team wins.
b) Determine the expected number of matches until the game terminates.

Reward:

a) Successful presentation of a valid solution to the problems above for the standard game in lecture for Math 3215 by the end of the semester will result in a payment of $15.00.

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