

# Benjamin Skelton MATH 3215 Poker Project

## Written Materials

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### 1 Introduction

In a standard game of poker. Players are handed a hand of cards containing 5 random cards from a standard deck. The standard deck contains 52 cards, there are 13 ranks of cards, and each rank can belong to 1 of 4 possible suits.

Depending on the cards in the player's hand, that hand can be designated one of ten possible ranks of varying value. A more "valuable" hand can beat a hand of less value. Ordered from least value to most value, the ten possible ranks of poker hands are

1. No pair
2. One pair
3. Two pair
4. Three of a kind
5. Straight
6. Flush
7. Full house
8. Four of a kind
9. Straight flush
10. Royal flush

### 2 Probability of being dealt each Poker Hand

Overall, if one is given 5 cards from a deck of 52 cards and the order in which cards are dealt does not matter, then there are  $\binom{52}{5}$  possible distinct hands. As a result, the odds of getting any particular poker hand are  $\frac{1}{\binom{52}{5}}$ .

To list the probability of getting each possible poker hand, we will define the uniform probability measure  $\pi : \wp(\text{Set of all possible poker hands}) \rightarrow [0, 1]$  such that  $\pi(\text{Set of } x \text{ possible poker hands}) = \frac{x}{\binom{52}{5}}$ .

## 2.1 No pair

Five cards of different ranks, the five cards cannot be of sequential rank nor can all five cards be of the same suit. To create a no pair hand:

1. Choose five different ranks for the five cards to take on.  $\binom{13}{5}$  possible choices.
2. Exclude those rank choices that would result in the five cards being of sequential rank. There are  $\binom{10}{1}$  possible rank sequences.
3. Choose one suit for the first card to take on.  $\binom{4}{1}$  possible choices.
4. Choose one suit for the second card to take on.  $\binom{4}{1}$  possible choices.
5. Choose one suit for the third card to take on.  $\binom{4}{1}$  possible choices.
6. Choose one suit for the fourth card to take on.  $\binom{4}{1}$  possible choices.
7. Choose one suit for the fifth card to take on.  $\binom{4}{1}$  possible choices.
8. Exclude those suit choices that would result in all five cards being of the same rank. There are  $\binom{4}{1}$  such suit choices.

Using the rule of multiplication, one can see that there are  $(\binom{13}{5} - \binom{10}{1}) * (\binom{4}{1}^5 - \binom{4}{1})$  possible no pair hands.

## 2.2 One pair

Two cards of the same rank but different suits, the remaining three cards are all of different ranks. To create a One pair hand:

1. Choose the rank that the one-pair will take on. The two cards of the pair must be of the same rank.  $\binom{13}{1}$  possible choices.
2. Choose two suits for the cards of the one-pair to take on.  $\binom{4}{2}$  possible choices.
3. Choose three ranks for the remaining three cards to take on. The remaining three cards must all be different ranks, and none of them can take on the rank chosen by the one-pair cards.  $\binom{12}{3}$  possible choices.
4. Choose a suit for the first non-pair card.  $\binom{4}{1}$  possible choices.
5. Choose a suit for the second non-pair card.  $\binom{4}{1}$  possible choices.
6. Choose a suit for the third non-pair card.  $\binom{4}{1}$  possible choices.

Using the rule of multiplication, one can see that there are  $\binom{13}{1} * \binom{4}{2} * \binom{12}{3} * \binom{4}{1}^3$  possible one pair hands.

## 2.3 Two pair

Two pairs of cards of the same rank, but the pairs are of separate ranks, the remaining card must be of a third rank. To create a two pair hand:

1. Choose the two ranks that the two pairs will take on.  $\binom{13}{2}$  possible choices.
2. Choose the two suits that the first pair of cards will take on.  $\binom{4}{2}$  possible choices.

3. Choose the two suits that the second pair of cards will take on.  $\binom{4}{2}$  possible choices.
4. Choose the rank that the single non-pair card will take on. This card must be a different rank from the ranks taken on by the two pairs.  $\binom{11}{1}$  possible choices.
5. Choose a suit for the single non-pair card.  $\binom{4}{1}$  possible choices.

Using the rule of multiplication, one can see that there are  $\binom{13}{2} * \binom{4}{2}^2 * \binom{11}{1} * \binom{4}{1}$  possible two pair hands.

## 2.4 Three of a kind

Three cards of the same rank, plus two additional cards of different ranks. To create a three of a kind hand:

1. Choose one rank for the three same-rank cards to take on.  $\binom{13}{1}$  possible choices.
2. Choose three different suits for the three same-rank cards to take on.  $\binom{4}{3}$  possible choices.
3. Choose two different ranks for the remaining two cards to take on.  $\binom{12}{2}$  possible choices.
4. Choose one suit for the first non-triplet card.  $\binom{4}{1}$  possible choices.
5. Choose one suit for the second non-triplet card.  $\binom{4}{1}$  possible choices.

Using the rule of multiplication, one can see that there are  $\binom{13}{1} * \binom{4}{3} * \binom{12}{2} * \binom{4}{1}^2$  possible three of a kind hands.

## 2.5 Straight

Five cards of sequential rank, the five cards cannot all be of the same suit because that would form a straight flush or royal flush hand. To create a straight hand.

1. Choose one rank for the lowest rank card in the sequence to take on.  $\binom{10}{1}$  possible choices.
2. Choose one suit for the first card in the sequence to take on.  $\binom{4}{1}$  possible choices.
3. Choose one suit for the second card in the sequence to take on.  $\binom{4}{1}$  possible choices.
4. Choose one suit for the third card in the sequence to take on.  $\binom{4}{1}$  possible choices.
5. Choose one suit for the fourth card in the sequence to take on.  $\binom{4}{1}$  possible choices.
6. Choose one suit for the fifth card in the sequence to take on.  $\binom{4}{1}$  possible choices.

Including the straight hands where all five cards are of the same suit, there are  $\binom{10}{1} * \binom{4}{1}^5$  possible straight hands. Excluding those hands that would form

a straight flush or royal flush hand, there are  $\binom{10}{1} * \binom{4}{1}^5 - (\binom{10}{1} * \binom{4}{1})$  possible straight hands.

There are  $(\binom{10}{1} * \binom{4}{1})$  possible hands where the five cards of sequential rank are all of the same hand. To create such a hand:

1. Choose one rank for the lowest rank card in the sequence to take on.  $\binom{10}{1}$  possible choices.
2. Choose one suit for all five cards in the sequence to take on.  $\binom{4}{1}$  possible choices.

## 2.6 Flush

Five cards all of the same suit, the five cards cannot be of sequential rank because that would form a straight flush or royal flush hand. To create a flush hand.

1. Choose five different ranks for the five cards to take on.  $\binom{13}{5}$  possible choices.
2. Choose one suit for all five cards to take on.  $\binom{4}{1}$  possible choices.

Including the flush hands where the five card are of sequential rank, there are  $\binom{13}{5} * \binom{4}{1}$  possible flush hands. Excluding those hands that would form a straight flush or royal flush hand, there are  $\binom{13}{5} * \binom{4}{1} - (\binom{10}{1} * \binom{4}{1})$  possible flush hands.

There are  $(\binom{10}{1} * \binom{4}{1})$  where the five cards of the same suit are also of sequential rank. To create such a hand:

1. Choose one rank for the lowest rank card in the sequence to take on.  $\binom{10}{1}$  possible choices.
2. Choose one suit for all five cards in the sequence to take on.  $\binom{4}{1}$  possible choices.

## 2.7 Full house

A triplet of cards of one rank, plus a pair of cards of a second rank. To create a full house hand:

1. Choose one rank for the triplet of cards to take on.  $\binom{13}{1}$  possible choices.
2. Choose three different suits for the triplet of cards to take on.  $\binom{4}{3}$  possible choices.
3. Choose one rank for the pair of cards to take on. This rank must be a different rank than that rank chosen for the triplet of cards.  $\binom{12}{1}$  possible choices.
4. Choose two different suits for the pair of cards to take on.  $\binom{4}{2}$  possible choices.

Using the rule of multiplication, one can see that there are  $\binom{13}{1} * \binom{4}{3} * \binom{12}{1} * \binom{4}{2}$  possible full house hands.

## 2.8 Four of a kind

Four cards of the same rank, plus one additional card of a different rank. To create a four of a kind hand:

1. Choose one rank for the four same-rank cards to take on.  $\binom{13}{1}$  possible choices.
2. Choose four different suits for the four same-rank cards to take on.  $\binom{4}{4}$  possible choices.
3. Choose one different rank for the one non-quadruplet card to take on.  $\binom{12}{1}$  possible choices.
4. Choose one suit for the one non-quadruplet card to take on.  $\binom{4}{1}$  possible choices.

Using the rule of multiplication, one can see that there are  $\binom{13}{1} * \binom{4}{4} * \binom{12}{1} * \binom{4}{1}$  possible four of a kind hands.

## 2.9 Straight flush

Five cards of sequential rank all of the same suit, the cards cannot take on the sequential ranks 10, Jack, Queen, King, and Ace because that would form a royal flush hand. To create a straight flush hand:

1. Choose one rank for the lowest rank card in the sequence to take on.  $\binom{10}{1}$  possible choices.
2. Choose one suit for the five cards to take on.  $\binom{4}{1}$  possible choices.

Including royal flush hands as a type of straight flush hand, there are  $\binom{10}{1} * \binom{4}{1}$  possible straight flush hands. Excluding royal flush hands from the set of possible straight flush hands, there are  $\binom{10}{1} * \binom{4}{1} - \binom{4}{1}$  possible hands.

## 2.10 Royal flush

A hand containing cards of the sequential ranks 10, Jack, Queen, King, and Ace, all cards are of the same suit. To create a royal flush hand:

1. Choose one suit for the five cards to take on.  $\binom{4}{1}$  possible choices.

Using the rule of multiplication, one can see that there are  $\binom{4}{1}$  possible royal flush hands.

## 2.11 Table of Probabilities

Here is a table showing the possible hands, the number of possible ways to get each hand, and their probability of getting each hand expressed both as a fraction, and as a percentage.

Table 1: Table of Poker Hand Probabilities

Hand Type	Possible Ways to get hand	Probability Fraction	Percentage Chance
No pair	$((\binom{13}{5} - \binom{10}{1}) * ((\binom{4}{1})^5 - \binom{4}{1}))$	$\frac{((\binom{13}{5} - \binom{10}{1}) * ((\binom{4}{1})^5 - \binom{4}{1}))}{\binom{52}{5}}$	50.12%
One pair	$\binom{13}{1} * \binom{4}{2} * \binom{12}{3} * \binom{4}{1}^3$	$\frac{\binom{13}{1} * \binom{4}{2} * \binom{12}{3} * \binom{4}{1}^3}{\binom{52}{5}}$	42.26%
Two pair	$\binom{13}{2} * \binom{4}{2}^2 * \binom{11}{1} * \binom{4}{1}$	$\frac{\binom{13}{2} * \binom{4}{2}^2 * \binom{11}{1} * \binom{4}{1}}{\binom{52}{5}}$	4.75%
Three of a kind	$\binom{13}{1} * \binom{4}{3} * \binom{12}{2} * \binom{4}{1}^2$	$\frac{\binom{13}{1} * \binom{4}{3} * \binom{12}{2} * \binom{4}{1}^2}{\binom{52}{5}}$	2.11%
Straight	$\binom{10}{1} * \binom{4}{1}^5 - ((\binom{10}{1}) * \binom{4}{1})$	$\frac{\binom{10}{1} * \binom{4}{1}^5 - ((\binom{10}{1}) * \binom{4}{1})}{\binom{52}{5}}$	0.39%
Flush	$\binom{13}{5} * \binom{4}{1} - ((\binom{10}{1}) * \binom{4}{1})$	$\frac{\binom{13}{5} * \binom{4}{1} - ((\binom{10}{1}) * \binom{4}{1})}{\binom{52}{5}}$	0.20%
Full house	$\binom{13}{1} * \binom{4}{3} * \binom{12}{1} * \binom{4}{2}$	$\frac{\binom{13}{1} * \binom{4}{3} * \binom{12}{1} * \binom{4}{2}}{\binom{52}{5}}$	0.14%
Four of a kind	$\binom{13}{1} * \binom{4}{4} * \binom{12}{1} * \binom{4}{1}$	$\frac{\binom{13}{1} * \binom{4}{4} * \binom{12}{1} * \binom{4}{1}}{\binom{52}{5}}$	0.024%
Straight flush	$\binom{10}{1} * \binom{4}{1} - ((\binom{4}{1}))$	$\frac{\binom{10}{1} * \binom{4}{1} - ((\binom{4}{1}))}{\binom{52}{5}}$	0.0014%
Royal flush	$\binom{4}{1}$	$\frac{\binom{4}{1}}{\binom{52}{5}}$	0.00015%

### 3 Probability of your opponent being dealt a better hand

Now that we have determined the probabilities of being dealt each hand, we must now answer a more important question. Namely, given that we know what kind of hand we are dealt, what is the probability that the opponent has a deck of a higher rank than the player as well as the probability that our opponent will have the same type of hand as ours, leading to a tie.

Determining these probabilities is not as simple as calculating the chance being dealt each hand from a standard deck, because we already know which 5 cards we have, which our opponent can no longer be randomly dealt, changing their odds of getting certain hands. The five cards we have cannot be obtained by our opponent, so these five cards can be considered “blocked”, there are  $C(47,5)$  or 1,533,939 possible hands our opponent can make using the 47 remaining cards. This can be seen as removing 5 random cards from the deck and then having to calculate probabilities based on the remaining cards. In addition, the number of possible ways that our opponent can form each kind of hand will vary on what cards the player currently has.

It would be extremely difficult to analyze the probabilities associated with all  $C(52, 5)$  possible hands you could have, so I have instead opted to show an example hand for each kind of hand, and then determine the probabilities for winning, losing, and getting a tie based on that example hand. This will not show all of the possible probabilities, but it should give an idea of your odds of winning given that you know what kind of hand you have.

**NOTE: I am about to go into an excruciating amount of detail on how to calculate the number of possible hands that your opponent can make. Unless you want to read several pages worth of calculations, I would recommend skipping to Section 3.11 on page 65 to see the table of probabilities.**

#### 3.1 Royal flush

Suppose you have the hand  $10h, Jh, Qh, Kh, Ah$  (10 of hearts, Jack of hearts, Queen of hearts, King of hearts, and Ace of hearts)

Number of hands that beat this hand: 0, a royal flush is the best kind of hand, so it is impossible to create a better hand.

Number of hands that tie with this hand (i.e., number of remaining possible royal flush hands): Your hand has “blocked” the possibility of creating a royal flush using cards from the hearts suit, leaving just 3 possible suits left. Therefore, there are 3 possible royal flush cards.

Royal flush probability tables:

Number of hands that beat yours	Chance of being beaten
None: 0	$\frac{0}{1,533,939} = 0\%$
Number of hands that tie with yours	Chance of being tied
Royal flush: 3	$\frac{3}{1,533,939} = 0.00020\%$
Chance of winning	
$100\% - 0\% - 0.00020\% = 99.9998\%$	

### 3.2 Straight flush

Suppose you have the hand  $6c, 7c, 8c, 9c, 10c$  (6 of clubs, 7 of clubs, 8 of clubs, 9 of clubs, 10 of clubs)

Number of hands that beat this hand:

A royal flush can beat this hand. Your hand has the card 10 of clubs, preventing your opponent from creating a royal flush using cards from the clubs suit, leaving just 3 possible royal flush hands.

Number of hands that tie with this hand:

A straight flush hand can tie with this hand.

If your opponent wants to create a straight flush using cards from the hearts, spades, or diamonds suits, then they have 3 choices of suit, and 9 choices of straights for each suit (9 is the highest possible card that can act as the lowest-rank card in the sequence), leading to  $(3 * 9 = 27)$  possible straight flush hands from different suits.

If your opponent wants to create a straight flush using cards from the clubs suit, then the only possible straight flush hand (not royal flush) is  $1c, 2c, 3c, 4c, 5c$ . Overall, there are  $(27 + 1 = 28)$  possible ways for your opponent to form a straight flush hand.

Straight flush probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 3	$\frac{3}{1,533,939} = 0.00020\%$
Total: 3	

Number of hands that tie with yours	Chance of being tied
Straight flush: 28	$\frac{28}{1,533,939} = 0.0018\%$
Chance of winning	
$100\% - 0.00020\% - 0.0018\% = 99.998\%$	

### 3.3 Four of a kind

Suppose that you have the hand 5c, 5s, 5d, 5h, 2d (5 of clubs, 5 of spades, 5 of diamonds, 5 of hearts, and 2 of diamonds)

Number of hands that can beat this hand:

A royal flush hand can beat yours, none of your cards are of the ranks “10”, “J”, “Q”, “K”, or “A”, so there are 4 possible royal flush hands.

A straight flush hand can beat yours, your sequence of cards “blocks” 20 possible straight flush hands (cards of rank 1-5 cannot act as the lowest rank card in the sequence because the “5” rank is blocked for all 4 suits, 5 sequences are blocked for each of the 4 suits, leading to 20 hands being blocked total). This results in  $(36 - 20 = 16)$  possible ways for your opponent to create a straight flush hand.

Number of hands that can tie with yours:

A four of a kind hand can tie with yours, to create a four of a kind hand:

1. Choose a rank aside from “2” or “5”, rank “5” cannot be used because all four “5” cards have been dealt, “2” cannot be used because one of the four “2” cards has been dealt already. This leaves 11 possible choices for ranks.

2. Choose four of four suits for the four same-rank cards.  $(C(4, 4) = 1)$  choices.

3. Choose one additional card. There are  $(47 - 4 = 43)$  remaining choices for cards.

Using the rule of multiplication, one can see that there are  $(11 * C(4, 4) * 43 = 11 * 1 * 43 = 473)$  possible ways for your opponent to create a four of a kind hand.

Four of a kind probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 4 Straight Flush: 20 Total: 24	$\frac{24}{1,533,939} = 0.0016\%$
Number of hands that tie with yours	Chance of being tied
Four of a kind: 473	$\frac{473}{1,533,939} = 0.031\%$
Chance of winning	
$100\% - 0.0016\% - 0.0031\% = 99.9953\%$	

### 3.4 Full house

Suppose that you have the hand  $9c, 9d, 9h, 4h, 4s$  (9 of clubs, 9 of diamonds, 9 of hearts, 4 of hearts, and 4 of spades)

Number of hands that can beat yours:

A royal flush hand can beat yours. Because the 9 of clubs, 9 of diamonds, and 9 of hearts are already blocked, a royal flush of spades is the only possible royal flush hand remaining, resulting in 1 possible way for your opponent to create a royal flush hand.

A straight flush hand can beat yours. The 9 of hearts and 4 of hearts are blocked, leaving no way to build a straight flush of hearts. The 4 of spades is blocked, making sequences where the lowest rank card is of rank 1-4 impossible, this results in  $(9 - 4 = 5)$  possible straight flush of spades hands. The 9 of diamonds is blocked, making 8 the highest possible highest-rank card in any sequence, which likewise makes 4 the highest possible lowest-rank card in any sequence, resulting in 4 possible straight flush of diamonds hands. The same logic applies to the suit of clubs, resulting in 4 possible straight flush of clubs hands. Overall, there are  $(5 + 4 + 4 = 13)$  possible ways for your opponent to create a straight flush hand.

A four of a kind hand can beat yours, to create a four of a kind hand:

1. Choose a rank besides "4" or "9". Ranks "4" and "9" are already blocked by your full house hand. This leaves 11 possible ranks.

2. Choose four of four suits for the four same-rank cards.  $C(4, 4)$  possible choices.

3. Choose one additional card.  $(47 - 4 = 43)$  possible choices.

Using the rule of multiplication, one can see that there are  $(11 * 1 * 43 = 473)$  possible ways for your opponent to create a four of a kind hand.

Number of hands that can tie with yours:

A full house hand can tie with yours. The triplet of same-rank cards cannot be of the ranks “4”, or “9”. To create a full house hand:

Method 1: The triplet of same-rank cards is not of the ranks “4” or “9”. The duo of same-rank cards is not of the ranks “4” or “9”:

1. For the triplet of same-rank cards, choose a rank aside from “4” or “9”. 11 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $C(4, 3) = 4$  possible choices.

3. For the duo of same-rank cards, choose a rank aside from “4”, “9”, or the rank chosen for step 1. 10 possible choices.

4. Choose two of four suits for the duo of same-rank cards.  $C(4, 2) = 6$  possible choices

By rule of multiplication,  $(11 * 4 * 10 * 6 = 2,640)$  possible choices.

Method 2: The triplet of same-rank cards is not of the ranks “4” or “9”. The duo of same-rank cards is of the rank “4”:

1. For the triplet of same-rank cards, choose a rank aside from “4” or “9”. 11 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $C(4, 3) = 4$  possible choices.

3. For the duo of same-rank cards, choose rank “4”. 1 possible choice.

4. Choose two of two suits for the duo of same-rank cards.  $C(2, 2) = 1$  possible choice.

By rule of multiplication,  $(11 * 4 * 1 * 1 = 44)$  possible choices.

Using the rule of addition, we can see that there are  $(2,640 + 44 = 2,684)$  possible ways for your opponent to create a full house deck.

Full house probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 1	$\frac{487}{1,533,939} = 0.032\%$
Straight Flush: 13	
Four of a kind: 473	
Total: 487	
Number of hands that tie with yours	Chance of being tied
Full house: 2,684	$\frac{2,684}{1,533,939} = 0.17\%$

Chance of winning
$100\% - 0.032\% - 0.17\% = 99.798\%$

### 3.5 Flush

Suppose that you have the hand  $2d, 4d, 6d, 10d, Qd$  (2 of diamonds, 4 of diamonds, 6 of diamonds, 10 of diamonds, and queen of diamonds).

Number of hands that can beat yours:

A royal flush hand can beat yours. A royal flush of diamonds is not possible, as  $10d$  and  $Qd$  are blocked, but royal flush hands of the three remaining suits are possible. Therefore, there are 3 ways for your opponent to create a royal flush hand.

A straight flush hand can beat yours. The ranks “2”, “4”, “6”, “10”, and “Q” of diamonds are all blocked, making it impossible to create a straight flush of diamonds. The other 3 suits are completely unblocked however, there are 9 possible straight flush hands per suit, so there are  $(9 * 3 = 27)$  possible ways for your opponent to create a straight flush hand.

A four of a kind hand can beat yours. To create a four of a kind hand:

1. Choose a rank besides “2”, “4”, “6”, “10”, or “Q”. 8 possible choices.
2. Choose four of four suits for the quadruplet of same-rank cards. 1 possible choice.
3. Choose one additional card to complete the hand.  $(47 - 4 = 43)$  possible choices.

By the rule of multiplication, there are  $(8 * 1 * 43 = 344)$  possible ways for your opponent to create a four of a kind hand.

A full house hand can beat yours. There are four methods your opponent can use to create a full house hand.

Method 1: Neither the triplet nor duo use “occupied” ranks

1. For the triplet of same-rank cards, choose a rank besides “2”, “4”, “6”, “10”, or “Q”. 8 possible choices.
2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.
3. For the duo of same-rank cards, choose a rank besides “2”, “4”, “6”, “10”, “Q” or the rank chosen in step 1. 7 possible choices.
4. Choose two of four suits for the duo of same-rank cards.  $(C(4,2) = 6)$  possible choices.

By the rule of multiplication, there are  $(8 * 4 * 7 * 6 = 1,344)$  possible hands using this method.

Method 2: The triplet uses an “occupied” rank, the duo does not

1. For the triplet of same-rank cards, choose one of the ranks “2”, “4”, “6”, “10”, or “Q”. 5 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose a rank aside from “2”, “4”, “6”, “10”, or “Q”. 8 possible choices.

4. Choose two of four suits for the duo of same-rank cards.  $(C(4,2) = 6)$  possible choices.

By the rule of multiplication, there are  $(5 * 1 * 8 * 6 = 240)$  possible hands using this method.

Method 3: The duo uses an “occupied” rank, the triplet does not

1. For the triplet of same-rank cards, choose a rank besides “2”, “4”, “6”, “10”, or “Q”. 8 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the duo of same-rank cards, choose one of the ranks “2”, “4”, “6”, “10”, or “Q”. 5 possible choices.

4. Choose two of three suits for the duo of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

By the rule of multiplication, there are  $(8 * 4 * 5 * 3 = 480)$  possible hands using this method.

Method 4: Both the triplet and duo use “occupied” ranks.

1. For the triplet of same-rank cards, choose one of the ranks “2”, “4”, “6”, “10”, or “Q”. 5 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose one of the ranks “2”, “4”, “6”, “10”, or “Q” aside from the rank chosen in step 1. 4 possible choices.

4. Choose two of three suits for the duo of same-rank cards.  $(C(3,2) = 3)$  possible choices.

By the rule of multiplication, there are  $(5 * 1 * 4 * 3 = 60)$  possible choices.

Using the rule of addition, we can see that there are  $(1,344 + 240 + 480 + 60 = 2,124)$  possible ways for your opponent to create a full house hand.

Number of hands that can tie with yours:

A flush hand can tie with yours.

If your opponent wants to create a flush of diamonds, the ranks “2”, “4”, “6”, “10”, and “Q” are blocked, making it impossible to create a straight flush of diamonds. Therefore, your opponent can create a flush of diamonds by simply choosing 5 of the 8 remaining ranks, resulting in  $(C(8, 5) = 56)$  possible flush of diamonds hands.

If your opponent wants to create a flush of clubs, spades or hearts:

1. Choose 13 of 5 rank cards.  $C(13, 5) = 1,287$  possible choices.

2. Choose one of three suits. 3 possible choices.

By the rule of multiplication, there are  $(1,287 * 3 = 3,861)$  possible flush hands of the remaining suits, but of these flush hands,  $(3 + 27 = 30)$  of them are straight flush or royal flush hands, so we need to subtract those hands to get a result of  $(3,861 - 30 = 3,831)$  possible flush hands of the three remaining suits.

Using the rule of addition, we can see that there are  $(56 + 3,831 = 3,887)$  possible ways for your opponent to create a flush hand.

Flush probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 3	$\frac{2,498}{1,533,939} = 0.16\%$
Straight Flush: 27	
Four of a kind: 344	
Full house: 2,124	
Total: 2,498	
Number of hands that tie with yours	Chance of being tied
Flush: 3,887	$\frac{3,887}{1,533,939} = 0.25\%$
Chance of winning	
$100\% - 0.16\% - 0.25\% = 99.59\%$	

### 3.6 Straight

Suppose that you have the hand  $3c, 4d, 5s, 6h, 7s$  (3 of clubs, 4 of diamonds, 5 of spades, 6 of hearts, and 7 of spades).

Number of hands that can beat yours:

A royal flush hand can beat yours. Because none of the ranks “10”, “J”, “Q”, “K”, or “A” are blocked. All four royal flush hands are possible.

A straight flush hand can beat yours. For a straight flush of clubs, any of the ranks 4-9 can act as the lowest-rank card in the sequence, leading to 6 possible straight flush of clubs hands. For a straight flush of spades, the ranks “5” and “7” are blocked, so only ranks “8” and “9” can act as the lowest-rank card in the straight flush sequence, leading to 2 possible straight flush of spades hands. For a straight flush of hearts, the rank “6” is blocked, so only ranks “A” and “7-9”

can act as the lowest-rank card in the sequence, leading to a total of 4 possible straight flush of hearts hands. For a straight flush of diamonds, the rank “4” is blocked, meaning that only ranks “5-9” can act as the lowest-rank card, leading to 5 possible straight flush of diamonds hands. By the rule of addition, there are  $(6 + 2 + 4 + 5 = 17)$  possible ways for your opponent to create a straight flush hand.

A four of a kind hand can beat yours. To create a four of a kind hand:

1. For the quadruplet of same-rank cards, choose a rank aside from “3”, “4”, “5”, “6”, or “7”. 8 possible choices.

2. Choose four of four suits for the quadruplet of same rank cards. 1 possible choice.

3. Choose one additional card to complete the hand.  $(47 - 4 = 43)$  possible choices.

By the rule of multiplication, there are  $(8 * 1 * 43 = 344)$  possible ways for your opponent to create a four of a kind hand.

A full house hand can beat yours. There are four methods your opponent can use to create a full house hand.

Method 1: Neither the triplet nor duo use “occupied” ranks

1. For the triplet of same-rank cards, choose a rank besides “3”, “4”, “5”, “6”, or “7”. 8 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the duo of same-rank cards, choose a rank besides “3”, “4”, “5”, “6”, “7” or the rank chosen in step 1. 7 possible choices.

4. Choose two of four suits for the duo of same-rank cards.  $(C(4,2) = 6)$  possible choices.

By the rule of multiplication, there are  $(8 * 4 * 7 * 6 = 1,344)$  possible hands using this method.

Method 2: The triplet uses an “occupied” rank, the duo does not

1. For the triplet of same-rank cards, choose one of the ranks “3”, “4”, “5”, “6”, or “7”. 5 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose a rank aside from “3”, “4”, “5”, “6”, or “7”. 8 possible choices.

4. Choose two of four suits for the duo of same-rank cards.  $(C(4,2) = 6)$  possible choices.

By the rule of multiplication, there are  $(5 * 1 * 8 * 6 = 240)$  possible hands using this method.

Method 3: The duo uses an “occupied” rank, the triplet does not

1. For the triplet of same-rank cards, choose a rank besides “3”, “4”, “5”, “6”, or “7”. 8 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the duo of same-rank cards, choose one of the ranks “3”, “4”, “5”, “6”, or “7”. 5 possible choices.

4. Choose two of three suits for the duo of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

By the rule of multiplication, there are  $(8 * 4 * 5 * 3 = 480)$  possible hands using this method.

Method 4: Both the triplet and duo use “occupied” ranks.

1. For the triplet of same-rank cards, choose one of the ranks “3”, “4”, “5”, “6”, or “7”. 5 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose one of the ranks “3”, “4”, “5”, “6”, or “7” aside from the rank chosen in step 1. 4 possible choices.

4. Choose two of three suits for the duo of same-rank cards.  $(C(3,2) = 3)$  possible choices.

By the rule of multiplication, there are  $(5 * 1 * 4 * 3 = 60)$  possible choices.

Using the rule of addition, we can see that there are  $(1,344 + 240 + 480 + 60 = 2,124)$  possible ways for your opponent to create a full house hand.

A flush hand can beat yours. To create a flush of clubs, there is one blocked clubs card, leading to  $(C(12, 5) = 792)$  possible flush of clubs hands. To create a flush of spades, there are two blocked spades cards, leading to  $(C(11, 5) = 462)$  possible flush of spades hands. To create a flush of hearts, there is one blocked hearts card, leading to  $(C(12, 5) = 792)$  possible flush of hearts hands. To create a flush of diamonds, there is one blocked diamonds card, leading to  $(C(12, 5) = 792)$  possible flush of diamonds hands.

By the rule of addition, there are  $(792 + 462 + 792 + 792 = 2,838)$  possible flush hands. We already know that  $(17 + 4 = 21)$  of these flush hands are also straight flush or royal flush hands, so there are only  $(2,838 - 21 = 2,817)$  possible ways for your opponent to create a flush hand.

Number of hands that can tie with yours:

A straight hand can beat yours. For straight hands containing cards of an “occupied” rank, we can use one of the three free suits of that rank.

To create a straight where “A” is the lowest rank (Rank Sequence “A”, “2”, “3”, “4”, “5”):

1. Choose one of four suits for the “A” card. 4 possible choices

2. Choose one of four suits for the “2” card. 4 possible choices.

3. Choose one of three suits for the “3” card. 3 possible choices.

4. Choose one of three suits for the “4” card. 3 possible choices.

5. Choose one of three suits for the “5” card. 3 possible choices.

By the rule of multiplication, there are  $(4 * 4 * 3 * 3 * 3 = 432)$  possible hands.

To create a straight where “2” is the lowest rank (Rank Sequence “2”, “3”, “4”, “5”, “6”):

1. Choose one of four suits for the “2” card. 4 possible choices.

2. Choose one of three suits for the “3” card. 3 possible choices.

3. Choose one of three suits for the “4” card. 3 possible choices.

4. Choose one of three suits for the “5” card. 3 possible choices.

5. Choose one of three suits for the “6” card. 3 possible choices.

By the rule of multiplication, there are  $(4 * 3 * 3 * 3 * 3 = 324)$  possible hands.

To create a straight where “3” is the lowest rank (Rank Sequence “3”, “4”, “5”, “6”, “7”):

1. Choose one of three suits for the “3” card. 3 possible choices.

2. Choose one of three suits for the “4” card. 3 possible choices.

3. Choose one of three suits for the “5” card. 3 possible choices.

4. Choose one of three suits for the “6” card. 3 possible choices.

5. Choose one of three suits for the “7” card. 3 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 3 * 3 * 3 = 243)$  possible choices.

To create a straight where “4” is the lowest rank (Rank Sequence “4”, “5”, “6”, “7”, “8”):

1. Choose one of three suits for the “4” card. 3 possible choices.

2. Choose one of three suits for the “5” card. 3 possible choices.

3. Choose one of three suits for the “6” card. 3 possible choices.

4. Choose one of three suits for the “7” card. 3 possible choices.

5. Choose one of four suits for the “8” card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 3 * 3 * 4 = 324)$  possible choices.

To create a straight where “5” is the lowest rank (Rank Sequence “5”, “6”, “7”, “8”, “9”):

1. Choose one of three suits for the “5” card. 3 possible choices.

2. Choose one of three suits for the “6” card. 3 possible choices.

3. Choose one of three suits for the “7” card. 3 possible choices.

4. Choose one of four suits for the “8” card. 4 possible choices.

5. Choose one of four suits for the “9” card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 3 * 4 * 4 = 432)$  possible choices.

To create a straight where “6” is the lowest rank (Rank Sequence “6”, “7”, “8”, “9”, “10”):

1. Choose one of three suits for the “6” card. 3 possible choices.
2. Choose one of three suits for the “7” card. 3 possible choices.
3. Choose one of four suits for the “8” card. 4 possible choices.
4. Choose one of four suits for the “9” card. 4 possible choices.
5. Choose one of four suits for the “10” card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 4 * 4 * 4 = 576)$  possible choices.

To create a straight where “7” is the lowest rank (Rank Sequence “7”, “8”, “9”, “10”, “J”):

1. Choose one of three suits for the “7” card. 3 possible choices.
2. Choose one of four suits for the “8” card. 4 possible choices.
3. Choose one of four suits for the “9” card. 4 possible choices.
4. Choose one of four suits for the “10” card. 4 possible choices.
5. Choose one of four suits for the “J” card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 4 * 4 * 4 * 4 = 768)$  possible choices.

To create a straight where “8” is the lowest rank (Rank Sequence “8”, “9”, “10”, “J”, “Q”):

1. Choose one of four suits for the “8” card. 4 possible choices.
2. Choose one of four suits for the “9” card. 4 possible choices.
3. Choose one of four suits for the “10” card. 4 possible choices.
4. Choose one of four suits for the “J” card. 4 possible choices.
5. Choose one of four suits for the “Q” card. 4 possible choices.

By the rule of multiplication, there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$  possible choices.

To create a straight where “9” is the lowest rank (Rank Sequence “9”, “10”, “J”, “Q”, “K”):

1. Choose one of four suits for the “9” card. 4 possible choices.
2. Choose one of four suits for the “10” card. 4 possible choices.
3. Choose one of four suits for the “J” card. 4 possible choices.
4. Choose one of four suits for the “Q” card. 4 possible choices.
5. Choose one of four suits for the “K” card. 4 possible choices.

By the rule of multiplication, there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$  possible choices.

To create a straight where “10” is the lowest rank (Rank Sequence “10”, “J”, “Q”, “K”, “A”):

1. Choose one of four suits for the “10” card. 4 possible choices.
2. Choose one of four suits for the “J” card. 4 possible choices.
3. Choose one of four suits for the “Q” card. 4 possible choices.
4. Choose one of four suits for the “K” card. 4 possible choices.
5. Choose one of four suits for the “A” card. 4 possible choices.

By the rule of multiplication, there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$  possible choices.

By summing all of choices from the possible methods to create a straight hand, there are  $(432 + 324 + 243 + 324 + 432 + 576 + 768 + 1,024 + 1,024 + 1,024 = 6,171)$  possible straight hands. We already know that  $(4 + 17 = 21)$  of these hands are straight flush or royal flush hands, so there are only  $(6,171 - 21 = 6,150)$  ways for your opponent to create a straight hand.

Straight probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 4 Straight Flush: 17 Four of a kind: 344 Full house: 2,124 Flush: 2,817 Total: 5,306	$\frac{5,306}{1,533,939} = 0.35\%$
Number of hands that tie with yours	Chance of being tied
Straight: 6,150	$\frac{6,150}{1,533,939} = 0.40\%$
Chance of winning	
$100\% - 0.35\% - 0.40\% = 99.25\%$	

### 3.7 Three of a kind

Suppose that you have the hand *7s, 7d, 7h, 3c, Ad* (7 of spades, 7 of diamonds, 7 of hearts, 3 of clubs, Ace of diamonds).

Number of hands that can beat yours:

A royal flush hand can beat yours. The ace of diamonds is blocked, leaving only 3 possible ways for your opponent to create a royal flush hand.

A straight flush hand can beat yours.

To create a straight flush of clubs, the rank “3” is blocked, meaning that only ranks “4-9” can act as the lowest-rank in the sequence, leading to 6 possible straight flush of clubs hands.

To create a straight flush of spades, the rank “7” is blocked, meaning that only ranks “A”, “2”, “8”, or “9” can act as the lowest-rank in the sequence, leading to 4 possible straight flush of spades hands.

To create a straight flush of hearts, the rank “7” is blocked, meaning that only ranks “A”, “2”, “8”, or “9” can act as the lowest-rank in the sequence, leading to 4 possible straight flush of hearts hands.

To create a straight flush of diamonds, the ranks “A” and “7” are blocked, meaning that only ranks “2”, “8”, or “9” can act as the lowest-rank in the sequence, leading to 3 possible straight flush of diamonds hands.

By the rule of addition, there are  $(6 + 4 + 4 + 3 = 17)$  possible ways for your opponent to create a straight flush hand.

A four of a kind hand can beat yours. To create a four of a kind hand:

1.For the quadruplet of same rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

2.Choose four of four suits for the quadruplet of same-rank cards. 1 possible choice.

3.Choose one additional card to complete the hand.  $(47 - 4 = 43)$  possible choices.

By the rule of multiplication, there are  $(10 * 1 * 43 = 430)$  possible ways for your opponent to create a four of a kind hand.

A full house hand can beat yours. There are four methods your opponent can use to create a full house hand:

Method 1: Neither the triplet nor duo are of the ranks “A” or “3”:

1.For the triplet of same-rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

2.Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3.For the duo of same-rank cards, choose a rank aside from “A”, “3”, “7”, or the rank chosen for step 1. 9 possible choices.

4.Choose two of four suits for the duo of same-rank cards.  $(C(4,2) = 6)$  possible choices.

By the rule of multiplication, there are  $(10 * 4 * 9 * 6 = 2,160)$  possible hands using this method.

Method 2: The triplet uses one of the ranks “A” or “3” while the duo does not:

1.For the triplet of same-rank cards, choose one of the ranks “A” or “3”. 2 possible choices

2.Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

4. Choose two of four suits for the duo of same-rank cards.  $(C(4,2) = 6)$  possible choices.

By the rule of multiplication, there are  $(2 * 1 * 10 * 6 = 120)$  possible hands using this method.

Method 3: The duo uses one of the ranks “A” or “3” while the triplet does not:

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the duo of same-rank cards, choose one of the ranks “A” or “3”. 2 possible choices.

4. Choose two of three suits for the duo of same-rank cards.  $(C(3,2) = 3)$  possible choices.

By the rule of multiplication, there are  $(10 * 4 * 2 * 3 = 240)$  possible hands using this method.

Method 4: Both the triplet and duo use the ranks “A” and “3”:

1. For the triplet of same-rank cards, choose one of the ranks “A” and “3”. 2 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose the one of the ranks “A” and “3” that was not selected in step 1. 1 possible choice.

4. Choose two of three suits for the duo of same-rank cards.  $(C(3,2) = 3)$  possible choices.

By the rule of multiplication, there are  $(2 * 1 * 1 * 3 = 6)$  possible hands using this method.

Using the rule of addition, we can see that there are  $(2,160 + 120 + 240 + 6 = 2,526)$  possible ways for your opponent to create a full house hand.

A flush hand can beat yours. To create a flush of clubs, the rank “3” is blocked, so there are  $(C(12, 5) = 792)$  possible flush of clubs hands. To create a flush of spades, the rank “7” is blocked, so there are  $(C(12, 5) = 792)$  possible flush of spades hands. To create a flush of hearts, the rank “7” is blocked, so there are  $(C(12, 5) = 792)$  possible flush of hearts hands. To create a flush of diamonds, the ranks “A” and “7” are blocked, so there are  $(C(11, 5) = 462)$  possible flush of diamonds hands.

By the rule of addition, we can see that there are  $(792 + 792 + 792 + 462 = 2,838)$  possible flush hands. We already know that  $(3 + 17 = 20)$  of these hands are straight flush or royal flush hands, so there are only  $(2,838 - 20 =$

2,818) possible ways for your opponent to create a flush hand.

A straight hand can beat yours. We will consider only those straight hands that use unblocked cards.

Considering those hands with the rank sequence "A", "2", "3", "4", "5", there are  $(3 * 4 * 3 * 4 * 4 = 576)$  possible choices. Considering those hands with the rank sequence "2", "3", "4", "5", "6", there are  $(4 * 3 * 4 * 4 * 4 = 768)$  possible choices. Considering those hands with the rank sequence "3", "4", "5", "6", "7", there are  $(3 * 4 * 4 * 4 * 1 = 192)$  possible choices. Considering those hands with the rank sequence "4", "5", "6", "7", "8", there are  $(4 * 4 * 4 * 1 * 4 = 256)$  possible choices.

Considering those hands with the rank sequence "5", "6", "7", "8", "9", there are  $(4 * 4 * 1 * 4 * 4 = 256)$  possible choices. Considering those hands with the rank sequence "6", "7", "8", "9", "10", there are  $(4 * 1 * 4 * 4 * 4 = 256)$  possible choices. Considering those hands with the rank sequence "7", "8", "9", "10", "J", there are  $(1 * 4 * 4 * 4 * 4 = 256)$  possible choices. Considering those hands with the rank sequence "8", "9", "10", "J", "Q", there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$  possible choices.

Considering those hands with the rank sequence "9", "10", "J", "Q", "K", there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$  possible choices. Considering those hands with the rank sequence "10", "J", "Q", "K", "A", there are  $(4 * 4 * 4 * 4 * 3 = 768)$  possible choices.

By the rule of addition, we can see that there are  $(576 + 768 + 192 + 256 + 256 + 256 + 256 + 1,024 + 1,024 + 768 = 5,376)$  possible straight hands. We already know that  $(3 + 17 = 20)$  of these hands will be straight flush or royal flush hands, so there are only  $(5,376 - 20 = 5,356)$  possible ways for your opponent to create a straight hand.

Number of hands that can tie with yours:

A three of a kind hand can tie with yours. There are 9 methods your opponent can use to create a three of a kind hand.

Method 1: The triplet does not use one of the ranks "A" or "3". The two remaining cards do not use any of the ranks "A", "3", or "7"

1. For the triplet of same-rank cards, choose a rank aside from "A", "3", or "7". 10 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the first remaining card, choose a rank aside from "A", "3", "7" or the rank chosen in step 1. 9 possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices

5. For the second remaining card, choose a rank aside from “A”, “3”, “7” or the ranks chosen in step 1 and step 3. 8 possible choices

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 9 * 4 * 8 * 4 = 46,080)$  possible hands. But the order in which the two remaining cards are picked/arranged does not matter, so every possible hand is “equivalent” to a hand with the same cards but with the order of the two remaining cards swapped. To get the true number of possible hands, we need to divide the number of possible hands by 2 to find that there are  $(46,080 / 2 = 23,040)$  possible unique hands using this method.

Method 2: The triplet does not use one of the ranks “A” or “3”. One of the two remaining cards uses one of the ranks “A” or “3”, while the other uses an unoccupied rank.

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the first remaining card, choose one of the ranks “A” or “3”. 2 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. For the second remaining card, choose a rank aside from “A”, “3”, “7”, or the rank chosen in step 1. 9 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 2 * 3 * 9 * 4 = 8,640)$  possible choices. We do not need to apply the division rule in this scenario because if you follow the steps in order, it is not possible to create a hand that uses the same cards but swaps the order of the two remaining cards.

Method 3: The triplet does not use one of the ranks “A” or “3”. The two remaining cards use the ranks “A” and “3”.

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the first remaining card, choose one of the ranks “A” or “3”. 2 possible choices.

4. Choose one of three suits for the first card. 3 possible choices.

5. For the second remaining card, choose the one of the ranks “A” and “3” that was not selected in step 3. 1 possible choice.

6. Choose one of three suits for the second card. 3 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 2 * 3 * 1 * 3 = 720)$  possible hands. After applying the division rule, there are  $(720 / 2 = 360)$  possible unique hands using this method.

Method 4: The triplet does not use one of the ranks “A” or “3”. One of two remaining cards uses the rank “7”, while the other uses an unoccupied rank.

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the first remaining card, choose the rank “7”. 1 possible choice.

4. Choose one of one suits for the first remaining card. 1 possible choice.

5. For the second remaining card, choose a rank aside from “A”, “3”, “7”, or the rank chosen for step 1. 9 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 1 * 1 * 9 * 4 = 1,440)$  possible hands. We do not need to apply the division rule in this scenario because if you follow the steps in order, it is not possible to create a hand that uses the same cards but swaps the order of the two remaining cards.

Method 5: The triplet does not use one of the ranks “A” or “3”. One of the remaining cards uses the rank “7”, while the other uses one of the ranks “A” or “3”.

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4,3) = 4)$  possible choices.

3. For the first remaining card, choose the rank “7”. 1 possible choice.

4. Choose one of one suit for the first remaining card. 1 possible choice.

5. For the second remaining card, choose one of the ranks “A” or “3”. 2 possible choices.

6. Choose one of three suits for the second remaining card. 3 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 1 * 1 * 2 * 3 = 240)$  possible hands. We do not need to apply the division rule in this scenario because if you follow the steps in order, it is not possible to create a hand that uses the same cards but swaps the order of the two remaining cards.

Method 6: The triplet does use one of the ranks “A” or “3”. The two remaining cards do not use any of the ranks “A”, “3”, or “7”.

1. For the triplet of same-rank cards, choose one of the ranks “A” or “3”. 2 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the first remaining card, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. For the second remaining card, choose a rank aside from “A”, “3”, “7”, or the rank chosen in step 3. 9 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 10 * 4 * 9 * 4 = 2,880)$  possible hands. After applying the division rule, there are  $(2,880 / 2 = 1,440)$  possible unique hands using this method.

Method 7: The triplet does use one of the ranks “A” or “3”. One of the two remaining cards is of the unused choice of ranks “A” and “3”, the other remaining card is of an unoccupied rank.

1.For the triplet of same-rank cards, choose one of the ranks “A” or “3”. 2 possible choices.

2.Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3.For the first remaining card, choose the one of the ranks “A” or “3” that was not chosen in step 1. 1 possible choice.

4.Choose one of three suits for the first remaining card. 3 possible choices.

5.For the second remaining card, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

6.Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 1 * 3 * 10 * 4 = 240)$  possible hands. We do not need to apply the division rule in this scenario because if you follow the steps in order, it is not possible to create a hand that uses the same cards but swaps the order of the two remaining cards.

Method 8: The triplet does use one of the ranks “A” or “3”. One of the two remaining cards is of the rank “7” while the other remaining card is of an unoccupied rank.

1.For the triplet of same-rank cards, choose one of the ranks “A” or “3”. 2 possible choices.

2.Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3.For the first remaining card, choose the rank “7”. 1 possible choice.

4.Choose one of one suit for the first remaining card. 1 possible choice.

5.For the second remaining card, choose a rank aside from “A”, “3”, or “7”. 10 possible choices.

6.Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 1 * 1 * 10 * 4 = 80)$  possible hands. We do not need to apply the division rule in this scenario because if you follow the steps in order, it is not possible to create a hand that uses the same cards but swaps the order of the two remaining cards.

Method 9: The triplet does use one of the ranks “A” or “3”. One of the two remaining cards is of the rank “7” while the other remaining card is of the unused choice of ranks “A” and “3”.

1.For the triplet of same-rank cards, choose one of the ranks “A” or “3”. 2 possible choices.

2.Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the first remaining card, choose the rank "7". 1 possible choice.
  4. Choose one of one suit for the first remaining card. 1 possible choice.
  5. For the second remaining card, choose the one of ranks "A" and "3" that was not selected in step 1. 1 possible choice.
  6. Choose one of three suits for the second remaining card. 3 possible choices.
- By the rule of multiplication, there are  $(2 * 1 * 1 * 1 * 1 * 3 = 6)$  possible hands. We do not need to apply the division rule in this scenario because if you follow the steps in order, it is not possible to create a hand that uses the same cards but swaps the order of the two remaining cards.

Using the rule of addition, we can see that there are  $(23,040 + 8,640 + 360 + 1,440 + 240 + 1,440 + 240 + 80 + 6 = 35,486)$  possible ways for your opponent to create a three of a kind hand.

Three of a kind probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 3	$\frac{11,150}{1,533,939} = 0.73\%$
Straight Flush: 17	
Four of a kind: 430	
Full house: 2,526	
Flush: 2,818	
Straight: 5,356	
Total: 11,150	
Number of hands that tie with yours	Chance of being tied
Three of a kind: 35,486	$\frac{35,486}{1,533,939} = 2.31\%$
Chance of winning	
$100\% - 0.73\% - 2.31\% = 96.96\%$	

### 3.8 Two pair

Suppose that you have the hand  $4d, 4h, Ks, Kh, Jc$  (4 of diamonds, 4 of hearts, king of spades, king of hearts, and jack of clubs).

Number of hands that can beat yours:

A royal flush hand can beat yours. The king of spades, king of diamonds, and jack of clubs are all blocked, making a royal flush of hearts the only possible way for your opponent to create a royal flush hand. Therefore, there is 1

possible way for your opponent to create a royal flush hand.

A straight flush hand can beat yours.

To create a straight flush of clubs, the “J” rank is blocked, so only the ranks “A-6” can act as the lowest-rank card in the sequence, leading to 6 possible straight flush of clubs hands.

To create a straight flush of spades, the “K” rank is blocked, so only the ranks “A-8” can act as the lowest-rank card in the sequence, leading to 8 possible straight flush of spades hands.

To create a straight flush of hearts, the ranks “4” and “K” are blocked, so only the ranks “5-8” can act as the lowest-rank card in the sequence, leading to 4 possible straight flush of hearts hands.

To create a straight flush of diamonds, the rank “4” is blocked, so only the ranks “5-9” can act as the lowest-rank card in the sequence, leading to 5 possible straight flush of diamonds hands.

By the rule of addition, there are  $(6 + 8 + 4 + 5 = 23)$  possible ways for your opponent to create a straight flush hand.

A four of a kind hand can beat yours. To create a four of a kind hand:

1. For the quadruplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.

2. Choose four of four suits for the quadruplet of same-rank cards. 1 possible choice.

3. Choose one additional card to complete the hand.  $(47 - 4 = 43)$  possible choices.

By the rule of multiplication, there are  $(10 * 1 * 43 = 430)$  possible ways for your opponent to create a four of a kind hand.

A full house hand can beat yours. The triplet cannot be of the ranks “4” or “K”, but the duo can be of any rank. There are 5 methods that your opponent can use to create a full house hand.

Method 1: The triplet is not of the rank “J”, the duo is not of the ranks “4”, “K”, or “J”:

1. For the triplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the duo of same-rank cards, choose a rank aside from “4”, “J”, “K”, or the rank chosen in step 1. 9 possible choices.

4. Choose two of three suits for the duo of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

By the rule of multiplication, there are  $(10 * 4 * 9 * 6 = 2,160)$  possible ways to create a full house hand using this method.

Method 2: The triplet is not of the rank “J”, the duo is of one of the ranks “4” or “K”:

1.For the triplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.

2.Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3.For the duo of same rank cards, choose one of the ranks “4” or “K”. 2 possible choices.

4.Choose two of two suits for the duo of same-rank cards. 1 possible choice.

By the rule of multiplication, there are  $(10 * 4 * 2 * 1 = 80)$  possible ways to create a full house hand using this method.

Method 3: The triplet is not of the rank “J”, the duo is of the rank “J”:

1.For the triplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.

2.Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3.For the duo of same-rank cards, choose the rank “J”. 1 possible choice.

4.Choose two of three suits for the duo of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

By the rule of multiplication, there are  $(10 * 4 * 1 * 3 = 120)$  possible ways to create a full house hand using this method.

Method 4: The triplet is of the rank “J”, the duo is not of the ranks “4”, “K”, or “J”:

1.For the triplet of same-rank cards, choose the rank “J”. 1 possible choice.

2.Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3.For the duo of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.

4.Choose two of four suits for the duo of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

By the rule of multiplication, there are  $(1 * 1 * 10 * 6 = 60)$  possible ways to create a full house hand using this method.

Method 5: The triplet is of the rank “J”, the duo is of one of the ranks “4” or “K”:

1.For the triplet of same-rank cards, choose the rank “J”. 1 possible choice.

2.Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3.For the duo of same-rank cards, choose one of the ranks “4” or “K”. 2 possible choices.

4.Choose two of two suits for the duo of same-rank cards. 1 possible choice.

By the rule of multiplication, there are  $(1 * 1 * 2 * 1 = 2)$  possible ways to create a full house hand using this method.

Using the rule of addition, we can see that there are  $(2,160 + 80 + 120 + 60 + 2 = 2,422)$  possible ways for your opponent to create a full house hand.

A flush hand can beat yours.

To create a flush of clubs, the rank “J” is blocked, so there are  $(C(12, 5) = 792)$  possible ways to create a flush of clubs hand.

To create a flush of spades, the rank “K” is blocked, so there are  $(C(12, 5) = 792)$  possible ways to create a flush of spades hand.

To create a flush of hearts, the ranks “4” and “K” are blocked, so there are  $(C(11, 5) = 462)$  possible ways to create a flush of hearts hand.

To create a flush of diamonds, the rank “4” is blocked, so there are  $(C(12, 5) = 792)$  possible ways to create a flush of diamonds hand.

By the rule of addition, there are  $(792 + 792 + 462 + 792 = 2,838)$  possible flush hands. We know that  $(1 + 23 = 24)$  of these hands are also straight flush or royal flush hands, so that leaves only  $(2,838 - 24 = 2,814)$  possible ways for your opponent to create a flush hand.

A straight hand can beat yours. We will consider only those straight hands that use unblocked cards.

Considering those straight hands with the rank sequence “A”, “2”, “3”, “4”, “5”, there are  $(4 * 4 * 4 * 2 * 4 = 512)$  possible choices. Considering those straight hands with the rank sequence “2”, “3”, “4”, “5”, “6”, there are  $(4 * 4 * 2 * 4 * 4 = 512)$  possible choices. Considering those straight hands with the rank sequence “3”, “4”, “5”, “6”, “7”, there are  $(4 * 2 * 4 * 4 * 4 = 512)$  possible choices. Considering those straight hands with the rank sequence “4”, “5”, “6”, “7”, “8”, there are  $(2 * 4 * 4 * 4 * 4 = 512)$  possible choices.

Considering those straight hands with the rank sequence “5”, “6”, “7”, “8”, “9”, there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$  possible choices. Considering those straight hands with the rank sequence “6”, “7”, “8”, “9”, “10”, there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$ . Considering those straight hands with the rank sequence “7”, “8”, “9”, “10”, “J”, there are  $(4 * 4 * 4 * 4 * 3 = 768)$  possible choices. Considering those straight hands with the rank sequence “8”, “9”, “10”, “J”, “Q”, there are  $(4 * 4 * 4 * 3 * 4 = 768)$  possible choices. Considering those straight hands with the rank sequence “9”, “10”, “J”, “Q”, “K”, there are  $(4 * 4 * 3 * 4 * 2 = 384)$  possible choices. Considering those straight hands with the rank sequence “10”, “J”, “Q”, “K”, “A”, there are  $(4 * 3 * 4 * 2 * 4 = 384)$

possible choices.

By the rule of addition, there are  $(512 + 512 + 512 + 512 + 1,024 + 1,024 + 768 + 768 + 384 + 384 = 6,400)$  possible straight hands. We know that  $(1 + 23 = 24)$  of these straight hands are also straight flush or royal flush hands, so there are only  $(6,400 - 24 = 6,376)$  possible ways for your opponent to create a straight hand.

A three of a kind hand can beat yours. The triplet cannot be of the ranks “4” or “K”. There are 6 methods that your opponent can use to create a three of a kind hand.

Method 1: The triplet is not of the rank “J”, the two remaining cards are not of the ranks “4”, “K”, or “J”:

1. For the triplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the two remaining cards, choose two ranks aside from “4”, “J”, “K”, or the rank chosen in step 1. Order does not matter.  $(C(9, 2) = 36)$  possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 36 * 4 * 4 = 23,040)$  possible ways to create a three of a kind hand using this method.

Method 2: The triplet is not of the rank “J”, one of the two remaining cards is of one of the ranks “4” or “K”, the other is of an unoccupied rank:

1. For the triplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the first remaining card, choose one of the ranks “4” or “K”. 2 possible choices.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose a rank aside from “4”, “J”, “K”, or the rank chosen in step 1. 9 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 2 * 2 * 9 * 4 = 5,760)$  possible ways to create a three of a kind hand using this method.

Method 3: The triplet is not of the rank “J”, one of the two remaining cards is of the ranks “4” or “K”, the other is of the rank “J”:

1. For the triplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the first remaining card, choose one of the ranks “4” or “K”. 2 possible choices.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose the rank “J”. 1 possible choice.

6. Choose one of three suits for the second remaining card. 3 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 2 * 2 * 1 * 3 = 480)$  possible ways to create a three of a kind hand using this method.

Method 4: The triplet is not of the rank “J”, the two remaining cards are of the ranks “4” and “K”:

1. For the triplet of same-rank cards, choose a rank aside from “4”, “J”, or “K”. 10 possible choices

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the first and second remaining cards, choose the two ranks “4” and “K”. Order does not matter.  $(C(2, 2) = 1)$  possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. Choose one of two suits for the second remaining card. 2 possible choices.

By the rule of multiplication, there are  $(10 * 4 * 1 * 2 * 2 = 160)$  possible ways to create a three of a kind hand using this method.

Method 5: The triplet is of the rank “J”, one of the two remaining card is of one of the ranks “4” or “K”, the other remaining card is of an unoccupied rank:

1. For the triplet of same-rank cards, choose the rank “J”. 1 possible choice.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the first remaining card, choose one of the ranks “4” or “K”. 2 possible choices.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(1 * 1 * 2 * 2 * 10 * 4 = 160)$  possible ways to create a three of a kind hand using this method.

Method 6: The triplet is of the rank “J”, the two remaining cards are of the ranks “4” and “K”:

1. For the triplet of same-rank cards, choose the rank “J”. 1 possible choice.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the two remaining cards, choose the ranks “4” and “K”. Order does not matter.  $(C(2, 2) = 1)$  possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. Choose one of two suits for the second remaining card. 2 possible choices.

By the rule of multiplication, there are  $(1 * 1 * 1 * 2 * 2 = 4)$  possible ways to create a three of a kind hand using this method.

Using the rule of addition, we can see that there are  $(23,040 + 5,760 + 480 + 160 + 160 + 4 = 29,604)$  possible ways for your opponent to create a three of a kind hand.

Number of hands that can tie with yours:

A two pair hand can tie with yours. There are 10 methods that your opponent can use to create a two pair hand.

Method 1: Neither of the two pairs are of the ranks “4”, “K”, or “J”, the one remaining card is not of the ranks “4”, “K”, or “J”:

1. For the two pairs, choose two ranks aside from “4”, “K”, or “J”. Order does not matter.  $(C(10, 2) = 45)$  possible choices.

2. Choose two of four suits for the first pair.  $(C(4, 2) = 6)$  possible choices.

3. Choose two of four suits for the second pair.  $(C(4, 2) = 6)$  possible choices.

4. For the one remaining card, choose a rank aside from “4”, “K”, “J”, or the two ranks chosen in step 1. 8 possible choices.

5. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(45 * 6 * 6 * 8 * 4 = 51,840)$  possible ways to create a two pair hand using this method.

Method 2: Neither of the two pairs are of the ranks “4”, “K”, or “J”, the one remaining card is of the ranks “4” or “K”:

1. For the two pairs, choose two ranks aside from “4”, “K”, or “J”. Order does not matter.  $(C(10, 2) = 45)$  possible choices.

2. Choose two of four suits for the first pair.  $(C(4, 2) = 6)$  possible choices.

3. Choose two of four suits for the second pair.  $(C(4, 2) = 6)$  possible choices.

4. For the one remaining card, choose one of the ranks “4” or “K”. 2 possible choices.

5. Choose one of two suits for the one remaining card. 2 possible choices.

By the rule of multiplication, there are  $(45 * 6 * 6 * 2 * 2 = 6,480)$  possible ways to create a two pair hand using this method.

Method 3: Neither of the two pairs are of the ranks “4”, “K”, or “J”, the one remaining card is of the rank “J”:

1. For the two pairs, choose two ranks aside from “4”, “K”, or “J”. Order does not matter.  $(C(10, 2) = 45)$  possible choices.

2. Choose two of four suits for the first pair.  $(C(4, 2) = 6)$  possible choices.

3. Choose two of four suits for the second pair.  $(C(4, 2) = 6)$  possible choices.

4. For the one remaining card, choose the rank “J”. 1 possible choice.

5. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(45 * 6 * 6 * 1 * 3 = 4,860)$  possible ways to create a two pair hand using this method.

Method 4: One of the two pairs is of the ranks “4” or “K”, the other pair is of an unoccupied rank, the one remaining card is also of an unoccupied rank:

1. For the first pair, choose one of the ranks “4” or “K”. 2 possible choices.
2. Choose two of two suits for the first pair. 1 possible choice.
3. For the second pair, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.
4. Choose two of four suits for the second pair.  $(C(4, 2) = 6)$  possible choices.
5. For the one remaining card, choose a rank aside from “4”, “J”, “K”, or the rank chosen in step 3. 9 possible choices.
6. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 10 * 6 * 9 * 4 = 4,320)$  possible ways to create a two pair hand using this method.

Method 5: One of the two pairs is of the ranks “4” or “K”, the other pair is of an unoccupied rank, the one remaining card is of the unselected rank of “4” and “K”:

1. For the first pair, choose one of the ranks “4” or “K”. 2 possible choices.
2. Choose two of two suits for the first pair. 1 possible choice.
3. For the second pair, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.
4. Choose two of four suits for the second pair.  $(C(4, 2) = 6)$  possible choices.
5. For the one remaining card, choose the one rank of “4” and “K” that was not selected in step 1. 1 possible choice.
6. Choose one of two suits for the one remaining card. 2 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 10 * 6 * 1 * 2 = 240)$  possible ways to create a two pair hand using this method.

Method 6: One of the two pairs is of the ranks “4” or “K”, the other pair is of an unoccupied rank, the one remaining card is of the rank “J”:

1. For the first pair, choose one of the ranks “4” or “K”. 2 possible choices.
2. Choose two of two suits for the first pair. 1 possible choice.
3. For the second pair, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.
4. Choose two of four suits for the second pair.  $(C(4, 2) = 6)$  possible choices.
5. For the one remaining card, choose the rank “J”. 1 possible choice.
6. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 10 * 6 * 1 * 3 = 360)$  possible ways to create a two pair hand using this method.

Method 7: The two pairs are of the ranks “4” and “K”, the one remaining card is of an unoccupied rank:

1. For the two pairs, choose the ranks “4” and “K”. Order does not matter.  $(C(2, 2) = 1)$  possible choice.
2. Choose two of two suits for the first pair. 1 possible choice.
3. Choose two of two suits for the second pair. 1 possible choice.

4. For the one remaining card, choose a rank aside from “4”, “J”, or “K”.  
10 possible choices.

5. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(1 * 1 * 1 * 10 * 4 = 40)$  possible ways to create a two pair hand using this method.

Method 8: The two pairs are of the ranks “4” and “K”, the one remaining card is of the rank “J”:

1. For the two pairs, choose the ranks “4” and “K”. Order does not matter.  $(C(2, 2) = 1)$  possible choice.

2. Choose two of two suits for the first pair. 1 possible choice.

3. Choose two of two suits for the second pair. 1 possible choice.

4. For the one remaining card, choose the rank “J”. 1 possible choice.

5. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(1 * 1 * 1 * 1 * 3 = 3)$  possible ways to create a two pair hand using this method.

Method 9: One of the pairs is of one of the ranks “4” and “K”, the other pair is of the rank “J”, the one remaining card is of an unoccupied rank:

1. For the first pair, choose one of the ranks “4” or “K”. 2 possible choices.

2. Choose two of two suits for the first pair. 1 possible choice.

3. For the second pair, choose the rank “J”. 1 possible choice.

4. Choose two of three suits for the second pair.  $(C(3, 2) = 3)$  possible choices.

5. For the one remaining card, choose a rank aside from “4”, “J”, or “K”. 10 possible choices.

6. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 1 * 3 * 10 * 4 = 240)$  possible ways to create a two pair hand using this method.

Method 10: One of the pairs is of one of the ranks “4” and “K”, the other pair is of the rank “J”, the one remaining card is of the unselected rank of “4” and “K”.

1. For the first pair, choose one of the ranks “4” or “K”. 2 possible choices.

2. Choose two of two suits for the first pair. 1 possible choice.

3. For the second pair, choose the rank “J”. 1 possible choice.

4. Choose two of three suits for the second pair.  $(C(3, 2) = 3)$  possible choices.

5. For the one remaining card, choose the one of the ranks “4” and “K” that was not selected in step 1. 1 possible choice.

6. Choose one of two suits for the one remaining card. 2 possible choices.

By the rule of multiplication, there are  $(2 * 1 * 1 * 3 * 1 * 2 = 12)$  possible ways to create a two pair hand using this method.

Using the rule of addition, we can see that there are  $(51,840 + 6,480 + 4,860 + 4,320 + 240 + 360 + 40 + 3 + 240 + 12 = 68,395)$  possible ways for your

opponent to create a two pair hand.

Two pair probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 1 Straight Flush: 23 Four of a kind: 430 Full house: 2,422 Flush: 2,814 Straight: 6,376 Three of a kind: 29,604 Total: 41,670	$\frac{41,670}{1,533,939} = 2.72\%$
Number of hands that tie with yours	Chance of being tied
Two pair: 68,395	$\frac{68,395}{1,533,939} = 4.46\%$
Chance of winning	
$100\% - 2.72\% - 4.46\% = 92.82\%$	

### 3.9 One pair

Suppose that you have the hand  $6c, 6s, 3d, 2h, 9s$  (6 of clubs, 6 of spades, 3 of diamonds, 2 of hearts, 9 of spades).

Number of hands that can beat yours:

A royal flush hand can beat yours. None of the ranks “10”, “J”, “Q”, “K”, “A” of any of the suits are blocked, so all 4 royal flush hands are possible.

A straight flush hand can beat yours.

To create a straight flush of clubs, the rank “6” is blocked, so only ranks “A” and “7-9” can act as the lowest-rank card in the sequence, leading to 4 possible straight flush of clubs hands.

To create a straight flush of spades, the ranks “6” and “9” are blocked, so only the rank “A” can act as the lowest-rank card in the sequence, leading to 1 possible straight flush of spades hand.

To create a straight flush of hearts, the rank “2” is blocked, so only the ranks “3-9” can act as the lowest-rank card in the sequence, leading to 7 possible straight flush of hearts hands.

To create a straight flush of diamonds, the rank “3” is blocked, so only the ranks “4-9” can act as the lowest-rank card in the sequence, leading to 6 possible straight flush of diamonds hands.

By the rule of addition, there are  $(4 + 1 + 7 + 6 = 18)$  possible ways for your opponent to create a straight flush hand.

A four of a kind hand can beat yours. To create a four of a kind hand:

1. For the quadruplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose four of four suits for the quadruplet of same-rank cards. 1 possible choice.

3. Choose one additional card to complete the hand.  $(47 - 4 = 43)$  possible choices.

By the rule of multiplication, there are  $(9 * 1 * 43 = 387)$  possible ways for your opponent to create a four of a kind hand.

A full house hand can beat yours. The triplet cannot be of the rank “6”, but the duo can be of any rank. There are 6 methods that your opponent can use to create a full house hand.

Method 1: Neither the triplet nor the duo of same-rank cards are of the ranks “2”, “3”, “6”, or “9”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the duo of same-rank cards, choose a rank aside from “2”, “3”, “6”, “9”, or the rank chosen in step 1. 8 possible choices.

4. Choose two of four suits for the duo of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

By the rule of multiplication, there are  $(9 * 4 * 8 * 6 = 1,728)$  possible ways to create a full house hand using this method.

Method 2: The triplet is of the ranks “2”, “3”, or “9”, the duo is of an unoccupied rank:

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

4. Choose two of four suits for the duo of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

By the rule of multiplication, there are ( $3 * 1 * 9 * 6 = 162$ ) possible ways to create a full house hand using this method.

Method 3: The triplet and the duo are both of the ranks “2”, “3”, and “9”:

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose one of the ranks “2”, “3”, or “9” that was not chosen in step 1. 2 possible choices.

4. Choose two of three suits for the duo of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

By the rule of multiplication, there are ( $3 * 1 * 2 * 3 = 18$ ) possible ways to create a full house hand using this method.

Method 4: The triplet is of the rank “2”, “3”, or “9”, the duo is of the rank “6”:

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose the rank “6”. 1 possible choice.

4. Choose two of two suits for the duo of same-rank cards. 1 possible choice.

By the rule of multiplication, there are ( $3 * 1 * 1 * 1 = 3$ ) possible ways to create a full house hand using this method.

Method 5: The triplet is not of the ranks “2”, “3”, “6” or “9”, the duo is of the ranks “2”, “3”, or “9”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the duo of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

4. Choose two of three suits for the duo of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

By the rule of multiplication, there are ( $9 * 4 * 3 * 3 = 324$ ) possible ways to create a full house hand using this method.

Method 6: The triplet is not of the ranks “2”, “3”, “6” or “9”, the duo is of the rank “6”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the duo of same-rank cards, choose the rank "6". 1 possible choice.

4. Choose two of two suits for the duo of same-rank cards. 1 possible choice.

By the rule of multiplication, there are  $(9 * 4 * 1 * 1 = 36)$  possible ways to create a full house hand using this method.

Using the rule of addition, we can see that there are  $(1,728 + 162 + 18 + 3 + 324 + 36 = 2,271)$  possible ways for your opponent to form a full house hand.

A flush hand can beat yours.

To create a flush of clubs, the rank "6" is blocked, so there are  $(C(12, 5) = 792)$  possible ways to create a flush of clubs hand.

To create a flush of spades, the ranks "6" and "9" are blocked, so there are  $(C(11, 5) = 462)$  possible ways to create a flush of spades hand.

To create a flush of hearts, the rank "2" is blocked, so there are  $(C(12, 5) = 792)$  possible ways to create a flush of hearts hand.

To create a flush of diamonds, the rank "3" is blocked, so there are  $(C(12, 5) = 792)$  possible ways to create a flush of diamonds hand.

By the rule of addition, there are  $(792 + 462 + 792 + 792 = 2,838)$  possible flush hands. We know that  $(18 + 4 = 22)$  of these hands will be straight flush or royal flush hands, so there are only  $(2,838 - 22 = 2,816)$  possible ways for your opponent to create a flush hand.

A straight hand can beat yours.

Considering those straight hands with the rank sequence "A", "2", "3", "4", "5", there are  $(4 * 3 * 3 * 4 * 4 = 576)$  possible choices. Considering those straight hands with the rank sequence "2", "3", "4", "5", "6", there are  $(3 * 3 * 4 * 4 * 2 = 288)$  possible choices. Considering those straight hands with the rank sequence "3", "4", "5", "6", "7", there are  $(3 * 4 * 4 * 2 * 4 = 384)$  possible choices. Considering those straight hands with the rank sequence "4", "5", "6", "7", "8", there are  $(4 * 4 * 2 * 4 * 4 = 512)$  possible choices.

Considering those straight hands with the rank sequence "5", "6", "7", "8", "9", there are  $(4 * 2 * 4 * 4 * 3 = 384)$  possible choices. Considering those straight hands with the rank sequence "6", "7", "8", "9", "10", there are  $(2 * 4 * 4 * 3 * 4 = 384)$  possible choices. Considering those straight hands with the rank sequence "7", "8", "9", "10", "J", there are  $(4 * 4 * 3 * 4 * 4 = 768)$  possible choices. Considering those straight hands with the rank sequence "8", "9", "10", "J", "Q", there are  $(4 * 3 * 4 * 4 * 4 = 768)$  possible choices.

Considering those straight hands with the rank sequence “9”, “10”, “J”, “Q”, “K”, there are  $(3 * 4 * 4 * 4 * 4 = 768)$  possible choices. Considering those straight hands with the rank sequence “10”, “J”, “Q”, “K”, “A”, there are  $(4 * 4 * 4 * 4 * 4 = 1,024)$  possible choices.

By the rule of addition, there are  $(576 + 288 + 384 + 512 + 384 + 384 + 768 + 768 + 768 + 1,024 = 5,856)$  possible ways for your opponent to form a straight hand. We know that  $(18 + 4 = 22)$  of these hands will be straight flush or royal flush hands, so there are only  $(5,856 - 22 = 5,834)$  possible ways for your opponent to create a straight hand.

A three of a kind hand can beat yours. The three of a kind cannot be of the rank “6”. There are 10 methods that your opponent can use to create a three of a kind hand.

Method 1: The triplet of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. The two remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.

3. For the two remaining cards, choose any two ranks aside from “2”, “3”, “6”, “9”, or the rank chosen in step 1. Order does not matter.  $(C(8, 2) = 28)$  possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(9 * 4 * 28 * 4 * 4 = 16,128)$  ways to create a three of a kind hand using this method.

Method 2: The triplet of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. One of the two remaining cards is of one of the ranks “2”, “3”, or “9”, while the other remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.

3. For the first remaining card, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. For the second remaining card, choose a rank aside from “2”, “3”, “6”, “9”, or the rank chosen in step 1. 8 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(9 * 4 * 3 * 3 * 8 * 4 = 10,368)$  possible ways to create a three of a kind hand using this method.

Method 3: The triplet of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. One of the two remaining cards is of the rank “6”, while the other remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.

3. For the first remaining card, choose the rank “6”. 1 possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose a rank aside from “2”, “3”, “6”, “9”, or the rank chosen in step 1. 8 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(9 * 4 * 1 * 2 * 8 * 4 = 2,304)$  possible ways to create a three of a kind hand using this method.

Method 4: The triplet of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. The two remaining cards are of two of the ranks of “2”, “3”, and “9”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.

3. For the two remaining cards, choose any two of the ranks “2”, “3”, and “9”. Order does not matter.  $(C(3, 2) = 3)$  possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

By the rule of multiplication, there are  $(9 * 4 * 3 * 3 * 3 = 972)$  possible ways to create a three of a kind hand using this method.

Method 5: The triplet of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. One of the two remaining cards is of the rank “6”, while the other is of one of the ranks of “2”, “3”, and “9”:

1. For the triplet of same-rank cards, choose a rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.

3. For the first remaining card, choose the rank “6”. 1 possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose one of the ranks “2”, “3”, and “9”. 3 possible choices.

6. Choose one of three suits for the second remaining card. 3 possible choices.

By the rule of multiplication, there are  $(9 * 4 * 1 * 2 * 3 * 3 = 648)$  possible ways to create a three of a kind hand using this method.

Method 6: The triplet of same-rank cards is of one of the ranks of “2”, “3”, and “9”. The two remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the two remaining cards, choose any two ranks aside from “2”, “3”, “6”, and “9”. Order does not matter.  $(C(9, 2) = 36)$  possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 1 * 36 * 4 * 4 = 1,728)$  possible ways to create a three of a kind hand using this method.

Method 7: The triplet of same-rank cards is of one of the ranks of “2”, “3”, and “9”. One of the two remaining cards is of one of the unselected ranks of “2”, “3”, and “9”, while the other remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the first remaining card, choose one rank of “2”, “3”, and “9”, that was not selected in step 1. 2 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. For the second remaining card, choose one rank aside from “2”, “3”, “6”, and “9”. 9 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 1 * 2 * 3 * 9 * 4 = 648)$  possible ways to create a three of a kind hand using this method.

Method 8: The triplet of same-rank cards is of one of the ranks of “2”, “3”, and “9”. One of the two remaining cards is of the rank “6”, while the other remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the first remaining card, choose the rank “6”. 1 possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose one rank aside from “2”, “3”, “6”, and “9”. 9 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 1 * 1 * 2 * 9 * 4 = 216)$  possible ways to create a three of a kind hand using this method.

Method 9: The triplet of same-rank cards is of one of the ranks of “2”, “3”, and “9”. The two remaining cards are of two of the unselected ranks of “2”, “3”, and “9”.

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the two remaining cards, choose any two of the ranks “2”, “3”, and “9” that was not selected in step 1. ( $C(2, 2) = 1$ ) possible choice.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

By the rule of multiplication, there are  $(3 * 1 * 1 * 3 * 3 = 27)$  possible ways to create a three of a kind hand using this method.

Method 10: The triplet of same-rank cards is of one of the ranks of “2”, “3”, and “9”. One of the two remaining cards is of the rank “6”, while the other remaining card is of one of the unselected ranks of “2”, “3”, and “9”.

1. For the triplet of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the first remaining card, choose the rank “6”. 1 possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose one of the ranks “2”, “3”, and “9” that was not selected in step 1. 2 possible choices.

6. Choose one of three suits for the second remaining card. 3 possible choices.

By the rule of multiplication, there are  $(3 * 1 * 1 * 2 * 2 * 3 = 36)$  possible ways to create a three of a kind hand using this method.

Using the rule of addition, we can see that there are  $(16,128 + 10,368 + 2,304 + 972 + 648 + 1,728 + 648 + 216 + 27 + 36 = 33,075)$  total possible ways for your opponent to form a three of a kind hand.

A two pair hand can beat yours. There are 13 methods that your opponent can use to create a two pair hand.

Method 1: Neither of the pairs of same-rank cards are of any of the ranks “2”, “3”, “6”, and “9”. The one remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the two pairs of same-rank cards, choose any two ranks aside from “2”, “3”, “6”, or “9”. Order does not matter. ( $C(9, 2) = 36$ ) possible choices.

2. Choose two of four suits for the first pair of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

3. Choose two of four suits for the second pair of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

4. For the one remaining card, choose one rank aside from “2”, “3”, “6”, “9”, or the two ranks chosen in step 1. 7 possible choices.

5. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(36 * 6 * 6 * 7 * 4 = 36,288)$  possible ways to create a two pair hand using this method.

Method 2: Neither of the pairs of same-rank cards are of any of the ranks “2”, “3”, “6”, and “9”. The one remaining card is of one of the ranks “2”, “3”, and “9”:

1. For the two pairs of same-rank cards, choose any two ranks aside from “2”, “3”, “6”, or “9”. Order does not matter.  $(C(9, 2) = 36)$  possible choices.

2. Choose two of four suits for the first pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

4. For the one remaining card, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

5. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(36 * 6 * 6 * 3 * 3 = 11,664)$  possible ways to create a two pair hand using this method.

Method 3: Neither of the pairs of same-rank cards are of any of the ranks “2”, “3”, “6”, and “9”. The one remaining card is of the rank “6”:

1. For the two pairs of same-rank cards, choose any two ranks aside from “2”, “3”, “6”, or “9”. Order does not matter.  $(C(9, 2) = 36)$  possible choices.

2. Choose two of four suits for the first pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

4. For the one remaining card, choose the rank “6”. 1 possible choice.

5. Choose one of two suits for the one remaining card. 2 possible choices.

By the rule of multiplication, there are  $(36 * 6 * 6 * 1 * 2 = 2,592)$  possible ways to create a two pair hand using this method.

Method 4: One of the pairs of same-rank cards is of one of the ranks of “2”, “3”, and “9”, the other pair is not of any of the ranks of “2”, “3”, “6”, and “9”. The one remaining card is not of any of the ranks of “2”, “3”, “6”, and “9”:

1. For the first pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the second pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

4. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

5. For the one remaining card, choose one rank aside from “2”, “3”, “6”, “9”, or the rank chosen in step 3. 8 possible choices.

6. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 9 * 6 * 8 * 4 = 15,552)$  possible ways to create a two pair hand using this method.

Method 5: One of the pairs of same-rank cards is of one of the ranks of “2”, “3”, and “9”, the other pair is not of any of the ranks of “2”, “3”, “6”, and “9”. The one remaining card is of one of the unselected ranks of “2”, “3”, and “9”:

1. For the first pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.
2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.
3. For the second pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.
4. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.
5. For the one remaining card, choose one of the ranks “2”, “3”, and “9” that was not selected in step 1. 2 possible choices.
6. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 9 * 6 * 2 * 3 = 2,916)$  possible ways to create a two pair hand using this method.

Method 6: One of the pairs of same-rank cards is of one of the ranks of “2”, “3”, and “9”, the other pair is not of any of the ranks of “2”, “3”, “6”, and “9”. The one remaining card is of one of the rank “6”:

1. For the first pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.
2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.
3. For the second pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.
4. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.
5. For the one remaining card, choose the rank “6”. 1 possible choice.
6. Choose one of two suits for the one remaining card. 2 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 9 * 6 * 1 * 2 = 972)$  possible ways to create a two pair hand using this method.

Method 7: One of the pairs of same-rank cards is of the rank “6”, the other pair is not of any of the ranks of “2”, “3”, “6”, and “9”. The one remaining card is not of any of the ranks of “2”, “3”, “6”, and “9”:

1. For the first pair of same-rank cards, choose the rank “6”. 1 possible choice.
2. Choose two of two suits for the first pair of same-rank cards. 1 possible choice.
3. For the second pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, and “9”. 9 possible choices.

4. Choose two of four suits for the second pair of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

5. For the one remaining card, choose one rank aside from “2”, “3”, “6”, “9”, and the rank chosen in step 3. 8 possible choices.

6. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are ( $1 * 1 * 9 * 6 * 8 * 4 = 1,728$ ) possible ways to create a two pair hand using this method.

Method 8: One of the pairs of same-rank cards is of the rank “6”, the other pair is not of any of the ranks of “2”, “3”, “6”, and “9”. The one remaining card is of one of the ranks of “2”, “3”, and “9”:

1. For the first pair of same-rank cards, choose the rank “6”. 1 possible choice.

2. Choose two of two suits for the first pair of same-rank cards. 1 possible choice.

3. For the second pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, and “9”. 9 possible choices.

4. Choose two of four suits for the second pair of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

5. For the one remaining card, choose one of the ranks “2”, “3”, and “9”. 3 possible choices.

6. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are ( $1 * 1 * 9 * 6 * 3 * 3 = 486$ ) possible ways to create a two pair hand using this method.

Method 9: The two pairs of same-rank cards are of two of the ranks of “2”, “3”, and “9”. The one remaining card is not of any of the ranks of “2”, “3”, “6”, and “9”:

1. For the two pairs of same-rank cards, choose any two ranks of “2”, “3”, and “9”. Order does not matter. ( $C(3, 2) = 3$ ) possible choices.

2. Choose two of three suits for the first pair of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

3. Choose two of three suits for the second pair of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

4. For the one remaining card, choose one rank aside from “2”, “3”, “6”, and “9”. 9 possible choices.

5. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are ( $3 * 3 * 3 * 9 * 4 = 972$ ) possible ways to create a two pair hand using this method.

Method 10: The two pairs of same-rank cards are of two of the ranks of “2”, “3”, and “9”. The one remaining card is of one of the unselected ranks of “2”, “3”, and “9”:

1. For the two pairs of same-rank cards, choose any two ranks of “2”, “3”, and “9”. Order does not matter. ( $C(3, 2) = 3$ ) possible choices.

2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.
  3. Choose two of three suits for the second pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.
  4. For the one remaining card, choose one of the ranks of “2”, “3”, and “9” that were not selected in step 1. 1 possible choice.
  5. Choose one of three suits for the one remaining card. 3 possible choices.
- By the rule of multiplication, there are  $(3 * 3 * 3 * 1 * 3 = 81)$  possible ways to create a two pair hand using this method.

Method 11: The two pairs of same-rank cards are of two of the ranks of “2”, “3”, and “9”. The one remaining card is of the rank “6”:

1. For the two pairs of same-rank cards, choose any two ranks of “2”, “3”, and “9”. Order does not matter.  $(C(3, 2) = 3)$  possible choices.
  2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.
  3. Choose two of three suits for the second pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.
  4. For the one remaining card, choose the rank “6”. 1 possible choice.
  5. Choose one of two suits for the one remaining card. 2 possible choices.
- By the rule of multiplication, there are  $(3 * 3 * 3 * 1 * 2 = 54)$  possible ways to create a two pair hand using this method.

Method 12: One of the pairs of same-rank cards is of the rank “6”, the other pair is of one of the ranks “2”, “3”, and “9”. The one remaining card is not of any of the ranks of “2”, “3”, “6”, and “9”:

1. For the first pair of same-rank cards, choose the rank “6”. 1 possible choice.
  2. Choose two of two suits for the first pair of same-rank cards. 1 possible choice.
  3. For the second pair of same-rank cards, choose one of the ranks “2”, “3”, and “9”. 3 possible choices.
  4. Choose two of three suits for the second pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.
  5. For the one remaining card, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.
  6. Choose one of four suits for the one remaining card. 4 possible choices.
- By the rule of multiplication, there are  $(1 * 1 * 3 * 3 * 9 * 4 = 324)$  possible ways to create a two pair hand using this method.

Method 13: One of the pairs of same-rank cards is of the rank “6”, the other pair is of one of the ranks “2”, “3”, and “9”. The one remaining card is of one of the unselected ranks of “2”, “3”, and “9”:

1. For the first pair of same-rank cards, choose the rank “6”. 1 possible choice.

2. Choose two of two suits for the first pair of same-rank cards. 1 possible choice.

3. For the second pair of same-rank cards, choose one of the ranks “2”, “3”, and “9”. 3 possible choices.

4. Choose two of three suits for the second pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

5. For the one remaining card, choose one of the ranks “2”, “3”, and “9” that was not selected in step 3. 2 possible choices.

6. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(1 * 1 * 3 * 3 * 2 * 3 = 54)$  possible ways to create a two pair hand using this method.

Using the rule of addition, we can see that there are  $(36,288 + 11,664 + 2,592 + 15,552 + 2,916 + 972 + 1,728 + 486 + 972 + 81 + 54 + 324 + 54 = 73,683)$  possible ways for your opponent to form a two pair hand.

Number of hands that can tie with yours:

A one pair hand can tie with yours. There are 17 methods that your opponent can use to create a one pair hand.

Method 1: The pair of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. The three remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. For the three remaining cards, choose any three ranks aside from “2”, “3”, “6”, “9”, or the rank chosen in step 1. Order does not matter.  $(C(8, 3) = 56)$  possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

6. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(9 * 6 * 56 * 4 * 4 * 4 = 193,536)$  possible ways to create a one pair hand using this method.

Method 2: The pair of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. One of the three remaining cards is of one of the ranks of “2”, “3”, and “9”. The other two remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.

2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. For the first remaining card, choose one of the ranks “2”, “3”, and “9”. 3 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.
  5. For the second and third remaining cards, choose any two ranks aside from "2", "3", "6", "9", or the rank chosen in step 1. Order does not matter.  $(C(8, 2) = 56)$  possible choices.
  6. Choose one of four suits for the second remaining card. 4 possible choices.
  7. Choose one of four suits for the third remaining card. 4 possible choices.
- By the rule of multiplication, there are  $(9 * 6 * 3 * 3 * 56 * 4 * 4 = 435,456)$  possible ways to create a one pair hand using this method.

Method 3: The pair of same-rank cards is not of any of the ranks "2", "3", "6", and "9". Two of the three remaining cards are of two of the ranks of "2", "3", and "9". The last remaining card is not of any of the ranks "2", "3", "6", and "9":

1. For the pair of same-rank cards, choose one rank aside from "2", "3", "6", or "9". 9 possible choices.
2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.
3. For the first and second remaining cards. Choose any two ranks of "2", "3", and "9". Order does not matter.  $(C(3, 2) = 3)$  possible choices.
4. Choose one of three suits for the first remaining card. 3 possible choices.
5. Choose one of three suits for the second remaining card. 3 possible choices.
6. For the last remaining card, choose one rank aside from "2", "3", "6", "9", or the rank chosen in step 1. 8 possible choices.
7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(9 * 6 * 3 * 3 * 3 * 8 * 4 = 46,656)$  possible ways to create a one pair hand using this method.

Method 4: The pair of same-rank cards is not of any of the ranks "2", "3", "6", and "9". The three remaining cards are of three of the ranks of "2", "3", and "9":

1. For the pair of same-rank cards, choose one rank aside from "2", "3", "6", or "9". 9 possible choices.
2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.
3. For the three remaining cards, choose three ranks of "2", "3", and "9". Order does not matter.  $(C(3, 3) = 1)$  possible choice.
4. Choose one of three suits for the first remaining card. 3 possible choices.
5. Choose one of three suits for the second remaining card. 3 possible choices.
6. Choose one of three suits for the third remaining card. 3 possible choices.

By the rule of multiplication, there are  $(9 * 6 * 1 * 3 * 3 * 3 = 1,458)$  possible ways to create a one pair hand using this method.

Method 5: The pair of same-rank cards is not of any of the ranks "2", "3", "6", and "9". One of the remaining cards is of the rank "6". The other two remaining cards are not of any of the ranks "2", "3", "6", and "9":

1. For the pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.
  2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.
  3. For the first remaining card, choose the rank “6”. 1 possible choice.
  4. Choose one of two suits for the first remaining card. 2 possible choices.
  5. For the second and third remaining cards, choose any two ranks aside from “2”, “3”, “6”, “9”, or the rank chosen in step 1. Order does not matter.  $(C(8, 2) = 28)$  possible choices.
  6. Choose one of four suits for the second remaining card. 4 possible choices.
  7. Choose one of four suits for the third remaining card. 4 possible choices.
- By the rule of multiplication, there are  $(9 * 6 * 1 * 2 * 28 * 4 * 4 = 48,384)$  possible ways to create a one pair hand using this method.

Method 6: The pair of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. One of the remaining cards is of the rank “6”. Another one of the remaining cards is of one of the ranks of “2”, “3”, and “9”. The last remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.
  2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.
  3. For the first remaining card, choose the rank “6”. 1 possible choice.
  4. Choose one of two suits for the first remaining card. 2 possible choices.
  5. For the second remaining card, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.
  6. Choose one of three suits for the second remaining card. 3 possible choices.
  7. For the third remaining card, choose one rank aside from “2”, “3”, “6”, “9”, or the rank chosen in step 1. 8 possible choices.
  8. Choose one of four suits for the third remaining card. 4 possible choices.
- By the rule of multiplication, there are  $(9 * 6 * 1 * 2 * 3 * 3 * 8 * 4 = 31,104)$  possible ways to create a one pair hand using this method.

Method 7: The pair of same-rank cards is not of any of the ranks “2”, “3”, “6”, and “9”. One of the remaining cards is of the rank “6”. The other two remaining cards are of two of the ranks of “2”, “3”, and “9”:

1. For the pair of same-rank cards, choose one rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.
2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.
3. For the first remaining card, choose the rank “6”. 1 possible choice.
4. Choose one of two suits for the first remaining card. 2 possible choices.
5. For the second and third remaining cards, choose any two of the ranks “2”, “3”, and “9”. Order does not matter.  $(C(3, 2) = 3)$  possible choices.
6. Choose one of three suits for the second remaining card. 3 possible choices.
7. Choose one of three suits for the third remaining card. 3 possible choices.

By the rule of multiplication, there are  $(9 * 6 * 1 * 2 * 3 * 3 * 3 = 2,916)$  possible ways to create a one pair hand using this method.

Method 8: The pair of same-rank cards is of one of the ranks of “2”, “3”, and “9”. None of the three remaining cards are of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose two of three suits for the pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the three remaining cards, choose any three ranks aside from “2”, “3”, “6”, or “9”.  $(C(9, 3) = 84)$  possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

6. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 84 * 4 * 4 * 4 = 48,384)$  possible ways to create a one pair hand using this method.

Method 9: The pair of same-rank cards is of one of the ranks of “2”, “3”, and “9”. One of the three remaining cards is of one of the unselected ranks of “2”, “3”, and “9”. The other two remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose two of three suits for the pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the first remaining card, choose one of the ranks “2”, “3” and “9” that was not selected in step 1. 2 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. For the second and third remaining card, choose any two ranks aside from “2”, “3”, “6”, and “9”. Order does not matter.  $(C(9, 2) = 36)$  possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 2 * 3 * 36 * 4 * 4 = 31,104)$  possible ways to create a one pair hand using this method.

Method 10: The pair of same-rank cards is of one of the ranks of “2”, “3”, and “9”. Two of the three remaining cards are of two of the unselected ranks of “2”, “3”, and “9”. The last remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose two of three suits for the pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the first and second remaining cards, choose any two ranks of “2”, “3”, and “9” that were not selected in step 1. Order does not matter.  $(C(2, 2) = 1)$

possible choice.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

6. For the third remaining card, choose one rank aside from “2”, “3”, “6”, and “9”. 9 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 1 * 3 * 3 * 9 * 4 = 2,916)$  possible ways to create a one pair hand using this method.

Method 11: The pair of same-rank cards is of one of the ranks of “2”, “3”, and “9”. One of the three remaining cards is of the rank “6”. The other two remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose two of three suits for the pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the first remaining card, choose the rank “6”. 1 possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second and third remaining cards, choose any two ranks aside from “2”, “3”, “6”, and “9”. Order does not matter.  $(C(9, 2) = 36)$  possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 1 * 2 * 36 * 4 * 4 = 10,368)$  possible ways to create a one pair hand using this method.

Method 12: The pair of same-rank cards is of one of the ranks of “2”, “3”, and “9”. One of the three remaining cards is of the rank “6”. Another one of the remaining cards is of one of the unselected ranks of “2”, “3”, and “9”. The last remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose two of three suits for the pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the first remaining card, choose the rank “6”. 1 possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second remaining card, choose one of the ranks “2”, “3”, and “9” that was not selected in step 1. 2 possible choices.

6. Choose one of three suits for the second remaining card. 3 possible choices.

7. For the third remaining card, choose one rank aside from “2”, “3”, “6”, and “9”. 9 possible choices.

8. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 1 * 2 * 2 * 3 * 9 * 4 = 3,888)$  possible ways to create a one pair hand using this method.

Method 13: The pair of same-rank cards is of one of the ranks of “2”, “3”, and “9”. One of the three remaining cards is of the rank “6”. The two other remaining cards are of two of the unselected ranks of “2”, “3”, and “9”:

1. For the pair of same-rank cards, choose one of the ranks “2”, “3”, or “9”. 3 possible choices.

2. Choose two of three suits for the pair of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

3. For the first remaining card, choose the rank “6”. 1 possible choice.

4. Choose one of two suits for the first remaining card. 2 possible choices.

5. For the second and third remaining cards, choose any two ranks of “2”, “3”, and “9” that were not selected in step 1. ( $C(2, 2) = 1$ ) possible choices.

6. Choose one of three suits for the second remaining card. 3 possible choices.

7. Choose one of three suits for the third remaining card. 3 possible choices.

By the rule of multiplication, there are  $(3 * 3 * 1 * 2 * 1 * 3 * 3 = 162)$  possible ways to create a one pair hand using this method.

Method 14: The pair of same-rank cards is of the rank “6”. The three remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose the rank “6”. 1 possible choice.

2. Choose two of two suits for the pair of same-rank cards. 1 possible choice.

3. For the three remaining cards, choose any three ranks aside from “2”, “3”, “6”, and “9”. Order does not matter. ( $C(9, 3) = 84$ ) possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

6. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(1 * 1 * 84 * 4 * 4 * 4 = 5,376)$  possible ways to create a one pair hand using this method.

Method 15: The pair of same-rank cards is of the rank “6”. One of the three remaining cards is of one of the ranks “2”, “3”, and “9”. The other two remaining cards are not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose the rank “6”. 1 possible choice.

2. Choose two of two suits for the pair of same-rank cards. 1 possible choice.

3. For the first remaining card, choose one of the ranks “2”, “3”, and “9”. 3 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. For the second and third remaining cards, choose any two ranks aside from “2”, “3”, “6”, and “9”. Order does not matter. ( $C(9, 2) = 36$ ) possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(1 * 1 * 3 * 3 * 36 * 4 * 4 = 5,184)$  possible ways to create a one pair hand using this method.

Method 16: The pair of same-rank cards is of the rank “6”. Two of the three remaining cards are of two of the ranks “2”, “3”, and “9”. The last remaining card is not of any of the ranks “2”, “3”, “6”, and “9”:

1. For the pair of same-rank cards, choose the rank “6”. 1 possible choice.
  2. Choose two of two suits for the pair of same-rank cards. 1 possible choice.
  3. For the first and second remaining cards, choose any two ranks of “2”, “3”, and “9”. Order does not matter.  $(C(3, 2) = 3)$  possible choices.
  4. Choose one of three suits for the first remaining card. 3 possible choices.
  5. Choose one of three suits for the second remaining card. 3 possible choices.
  6. For the third remaining card, choose any rank aside from “2”, “3”, “6”, or “9”. 9 possible choices.
  7. Choose one of four suits for the third remaining card. 4 possible choices.
- By the rule of multiplication, there are  $(1 * 1 * 3 * 3 * 3 * 9 * 4 = 972)$  possible ways to create a one pair hand using this method.

Method 17: The pair of same-rank cards is of the rank “6”. The three remaining cards are of three of the ranks of “2”, “3”, and “9”:

1. For the pair of same-rank cards, choose the rank “6”. 1 possible choice.
  2. Choose two of two suits for the pair of same-rank cards. 1 possible choice.
  3. For the three remaining cards, choose any three of the ranks “2”, “3”, and “9”. Order does not matter.  $(C(3, 3) = 1)$  possible choice.
  4. Choose one of three suits for the first remaining card. 3 possible choices.
  5. Choose one of three suits for the second remaining card. 3 possible choices.
  6. Choose one of three suits for the third remaining card. 3 possible choices.
- By the rule of multiplication, there are  $(1 * 1 * 1 * 3 * 3 * 3 = 27)$  possible ways to create a one pair hand using this method.

Using the rule of addition, we can see that there are  $(193,536 + 435,456 + 46,656 + 1,458 + 48,384 + 31,104 + 2,916 + 48,384 + 31,104 + 2,916 + 10,368 + 3,888 + 162 + 5,376 + 5,184 + 972 + 27 = 867,891)$  possible ways for your opponent to form a one pair hand.

One pair probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 4	$\frac{118,088}{1,533,939} = 7.70\%$
Straight Flush: 18	
Four of a kind: 387	
Full house: 2,271	
Flush: 2,816	
Straight: 5,834	
Three of a kind: 33,075	
Two pair: 73,683	
Total: 118,088	

Number of hands that tie with yours	Chance of being tied
One pair: 867,891	$\frac{867,891}{1,533,939} = 56.58\%$
Chance of winning	
$100\% - 7.70\% - 56.58\% = 35.72\%$	

### 3.10 No pair

Suppose that you have the hand *Ad, 3h, 6c, 8c, 10s* (Ace of diamonds, 3 of hearts, 6 of clubs, 8 of clubs, and 10 of spades).

Number of hands that can beat yours:

A royal flush hand can beat yours. The ace of diamonds and 10 of spades are blocked, meaning that only the royal flush of hearts and royal flush of clubs are possible. Therefore, there are 2 possible ways for your opponent to create a royal flush hand.

A straight flush hand can beat yours.

To create a straight flush of clubs the ranks “6” and “8” are blocked, so only the ranks “A” and “9” can act as the lowest-rank in the sequence. Therefore, there are 2 straight flush of clubs hands.

To create a straight flush of spades, the rank “10” is blocked, so only the ranks “A-5” can act as the lowest-rank in the sequence. Therefore, there are 5 straight flush of spades hands.

To create a straight flush of hearts, the rank “3” is blocked, so only the ranks “4-9” can act as the lowest ranks in the sequence. Therefore, there are 6 straight flush of hearts hands.

To create a straight flush of diamonds, the rank “A” is blocked, so only the ranks “2-9” can act as the lowest-rank in the sequence. Therefore, there are 8 straight flush of diamonds hands.

By the rule of addition, there are  $(2 + 5 + 6 + 8 = 21)$  possible ways to create a straight flush hand.

A four of a kind hand can beat yours. To create a four of a kind hand:

1. For the quadruplet of same-rank cards, choose a rank aside from “A”, “3”, “6”, “8”, or “10”. 8 possible choices.
2. Choose four suits for the quadruplet of same-rank cards. 1 possible choice.

3. Choose one additional card to complete the hand. ( $47 - 4 = 43$ ) possible choices.

By the rule of multiplication, there are ( $8 * 1 * 43 = 344$ ) possible ways for your opponent to create a four of a kind hand.

A full house hand can beat yours. There are 4 methods your opponent can use to create a full house hand.

Method 1: Neither the triplet nor the duo are of the ranks "A", "3", "6", "8", or "10":

1. For the triplet of same-rank cards, choose one rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the duo of same-rank cards, choose a rank aside from "A", "3", "6", "8", "10", or the rank chosen in step 1. 7 possible choices.

4. Choose two of four suits for the duo of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

By the rule of multiplication, there are ( $8 * 4 * 7 * 6 = 1,344$ ) possible ways to create a full house hand using this method.

Method 2: The triplet is of one of the ranks "A", "3", "6", "8", or "10", but the duo is not:

1. For the triplet of same-rank cards, choose one of the ranks "A", "3", "6", "8", or "10". 5 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose a rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

4. Choose two of four suits for the duo of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

By the rule of multiplication, there are ( $5 * 1 * 8 * 6 = 240$ ) possible ways to create a full house hand using this method.

Method 3: The duo is of one of the ranks "A", "3", "6", "8", or "10", but the triplet is not:

1. For the triplet of same-rank cards, choose one rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

2. Choose three of four suits for the triplet of same-rank cards. 4 possible choices.

3. For the duo of same-rank cards, choose one of the ranks "A", "3", "6", "8", or "10". 5 possible choices.

4. Choose two of three suits for the duo of same-ranks cards. 3 possible choices.

By the rule of multiplication, there are ( $8 * 4 * 5 * 3 = 480$ ) possible ways to create a full house hand using this method.

Method 4: Both the triplet and the duo are of the ranks “A”, “3”, “6”, “8”, or “10”:

1. For the triplet of same-rank cards, choose one of the ranks “A”, “3”, “6”, “8”, or “10”. 5 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the duo of same-rank cards, choose one of the ranks “A”, “3”, “6”, “8”, or “10” that was not chosen in step 1. 4 possible choices.

4. Choose two of three suits for the duo of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

By the rule of multiplication, there are ( $5 * 1 * 4 * 3 = 60$ ) possible ways to create a full house hand using this method.

Using the rule of addition, we can see that there are ( $1,344 + 240 + 480 + 60 = 2,124$ ) possible ways for your opponent to create a full house hand.

A flush hand can beat yours.

To create a flush of clubs, the ranks “6” and “8” are both blocked, so there are ( $C(11, 5) = 462$ ) possible ways to form a flush of clubs hand.

To create a flush of spades, the rank “10” is blocked, so there are ( $C(12, 5) = 792$ ) possible ways to form a flush of spades hand.

To create a flush of hearts, the rank “3” is blocked, so there are ( $C(12, 5) = 792$ ) possible ways to form a flush of hearts hand.

To create a flush of diamonds, the rank “A” is blocked, so there are ( $C(12, 5) = 792$ ) possible ways to form a flush of diamonds hand.

By the rule of addition, there are ( $462 + 792 + 792 + 792 = 2,838$ ) possible ways for your opponent to form a flush hand. We know that ( $2 + 21 = 23$ ) of these flush hands are straight flush or royal flush hands, so there are only ( $2,838 - 23 = 2,815$ ) possible ways for your opponent to form a flush hand.

A straight hand can beat yours.

Considering those straight hands with the rank sequence “A”, “2”, “3”, “4”, “5”, there are ( $3 * 4 * 3 * 4 * 4 = 576$ ) possible choices. Considering those straight hands with the rank sequence “2”, “3”, “4”, “5”, “6”, there are ( $4 * 3 * 4 * 4 * 3 = 576$ ) possible choices. Considering those straight hands with the rank sequence “3”, “4”, “5”, “6”, “7”, there are ( $3 * 4 * 4 * 3 * 4 = 576$ ) possible choices. Considering those straight hands with the rank sequence “4”, “5”, “6”, “7”, “8”, there are ( $4 * 4 * 3 * 4 * 3 = 576$ ) possible choices.

Considering those straight hands with the rank sequence “5”, “6”, “7”, “8”, “9”, there are  $(4 * 3 * 4 * 3 * 4 = 576)$  possible choices. Considering those straight hands with the rank sequence “6”, “7”, “8”, “9”, “10”, there are  $(3 * 4 * 3 * 4 * 3 = 432)$  possible choices. Considering those straight hands with the rank sequence “7”, “8”, “9”, “10” “J”, there are  $(4 * 3 * 4 * 3 * 4 = 576)$  possible choices. Considering those straight hands with the rank sequence “8”, “9”, “10”, “J”, “Q”, there are  $(3 * 4 * 3 * 4 * 4 = 576)$  possible choices. Considering those straight hands with the rank sequence “9”, “10”, “J”, “Q”, “K”, there are  $(4 * 3 * 4 * 4 * 4 = 768)$  possible choices. Considering those straight hands with the rank sequence “10”, “J”, “Q”, “K”, “A”, there are  $(3 * 4 * 4 * 4 * 3 = 576)$  possible choices.

Using the rule of addition, we can see that there are  $(576 + 576 + 576 + 576 + 576 + 432 + 576 + 576 + 768 + 576 = 5,808)$  possible ways to create a straight hand. We know that  $(2 + 21 = 23)$  of these straight hands are straight flush or royal flush hands, so there are only  $(5,808 - 23 = 5,785)$  possible ways for your opponent to form a straight hand.

A three of a kind hand can beat yours. There are 6 methods that your opponent can use to create a three of a kind hand.

Method 1: Neither the triplet of same-rank cards nor the two remaining cards are of the ranks “A”, “3”, “6”, “8”, or “10”:

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, “6”, “8”, or “10”. 8 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.

3. For the two remaining cards, choose two ranks aside from “A”, “3”, “6”, “8”, “10”, or the rank chosen in step 1. Order does not matter.  $(C(7, 2) = 21)$  possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(8 * 4 * 21 * 4 * 4 = 10,752)$  possible ways to create a three of a kind hand using this method.

Method 2: The triplet of same-rank cards is not of the ranks “A”, “3”, “6”, “8”, or “10”, one of the two remaining cards is of one of the ranks “A”, “3”, “6”, “8”, or “10”, but the other is not:

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, “6”, “8”, or “10”. 8 possible choices.

2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.

3. For the first remaining card, choose one of the ranks “A”, “3”, “6”, “8”, or “10”. 5 possible choices.

4. For the second remaining card, choose a rank aside from “A”, “3”, “6”, “8”, “10”, or the rank chosen in step 1. 7 possible choices.

5. Choose one of three suits for the first remaining card. 3 possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.  
 By the rule of multiplication, there are  $(8 * 4 * 5 * 7 * 3 * 4 = 13,440)$  possible ways to create a three of a kind hand using this method.

Method 3: The triplet of same-rank cards is not of the ranks “A”, “3”, “6”, “8”, or “10”, but both of the two remaining cards are of the ranks “A”, “3”, “6”, “8”, or “10”:

1. For the triplet of same-rank cards, choose a rank aside from “A”, “3”, “6”, “8”, or “10”. 8 possible choices.
  2. Choose three of four suits for the triplet of same-rank cards.  $(C(4, 3) = 4)$  possible choices.
  3. For the two remaining cards, choose any two ranks of the ranks “A”, “3”, “6”, “8”, and “10”. Order does not matter.  $(C(5, 2) = 10)$  possible choices.
  4. Choose one of three suits for the first remaining card. 3 possible choices.
  5. Choose one of three suits for the second remaining card. 3 possible choices.
- By the rule of multiplication, there are  $(8 * 4 * 10 * 3 * 3 = 2,880)$  possible ways to create a three of a kind hand using this method.

Method 4: The triplet of same-rank cards is of one of the ranks “A”, “3”, “6”, “8”, or “10”, but neither of the two remaining cards is of the ranks “A”, “3”, “6”, “8”, or “10”:

1. For the triplet of same-rank cards, choose one of the ranks “A”, “3”, “6”, “8”, or “10”. 5 possible choices.
  2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.
  3. For the two remaining cards, choose two ranks aside from “A”, “3”, “6”, “8”, or “10”. Order does not matter.  $(C(8, 2) = 28)$  possible choices.
  4. Choose one of four suits for the first remaining card. 4 possible choices.
  5. Choose one of four suits for the second remaining card. 4 possible choices.
- By the rule of multiplication, there are  $(5 * 1 * 28 * 4 * 4 = 2,240)$  possible ways to create a three of a kind hand using this method.

Method 5: The triplet of same-rank cards is of one of the ranks “A”, “3”, “6”, “8”, or “10”, one of the two remaining cards is of one of the unselected ranks of “A”, “3”, “6”, “8”, or “10”, but the other is not:

1. For the triplet of same-rank cards, choose one of the ranks “A”, “3”, “6”, “8”, or “10”. 5 possible choices.
2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.
3. For the first remaining card, choose one of the ranks “A”, “3”, “6”, “8”, or “10” that was not selected in step 1. 4 possible choices.
4. For the second remaining card, choose a rank aside from “A”, “3”, “6”, “8”, or “10”. 8 possible choices.
5. Choose one of three suits for the first remaining card. 3 possible choices.
6. Choose one of four suits for the second remaining card. 4 possible choices.

By the rule of multiplication, there are  $(5 * 1 * 4 * 8 * 3 * 4 = 1,920)$  possible ways to create a three of a kind hand using this method.

Method 6: The triplet of same-rank cards is of one of the ranks “A”, “3”, “6”, “8”, or “10”, the two remaining cards are both of the unselected ranks of “A”, “3”, “6”, “8”, or “10”:

1. For the triplet of same-rank cards, choose one of the ranks “A”, “3”, “6”, “8”, or “10”. 5 possible choices.

2. Choose three of three suits for the triplet of same-rank cards. 1 possible choice.

3. For the two remaining cards, choose any two of the ranks “A”, “3”, “6”, “8”, or “10” that was not selected in step 1.  $(C(4, 2) = 6)$  possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

By the rule of multiplication, there are  $(5 * 1 * 6 * 3 * 3 = 270)$  possible ways to create a three of a kind hand using this method.

Using the rule of addition, we can see that there are  $(10,752 + 13,440 + 2,880 + 2,240 + 1,920 + 270 = 31,502)$  possible ways for your opponent to form a three of a kind hand.

A two pair hand can beat yours. There are 6 methods that your opponent can use to create a two pair hand.

Method 1: Neither of the two pairs of same-rank cards are of the ranks “A”, “3”, “6”, “8”, or “10”. The one remaining card is not of the ranks “A”, “3”, “6”, “8”, or “10”:

1. For the two pairs of same-rank cards, choose any two ranks aside from “A”, “3”, “6”, “8”, or “10”. Order does not matter.  $(C(8, 2) = 28)$  possible choices.

2. Choose two of four suits for the first pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

4. For the one remaining card, choose a rank aside from “A”, “3”, “6”, “8”, “10”, or the two ranks chosen in step 1. 6 possible choices.

5. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(28 * 6 * 6 * 6 * 4 = 24,192)$  possible ways to create a two pair hand using this method.

Method 2: Neither of the two pairs of same-rank cards are of the ranks “A”, “3”, “6”, “8”, or “10”. The one remaining card is of one of the ranks “A”, “3”, “6”, “8”, or “10”:

1. For the two pairs of same-rank cards, choose any two ranks aside from “A”, “3”, “6”, “8”, or “10”. Order does not matter.  $(C(8, 2) = 28)$  possible choices.

2. Choose two of four suits for the first pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

4. For the one remaining card, choose one of the ranks "A", "3", "6", "8", or "10". 5 possible choices.

5. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(28 * 6 * 6 * 5 * 3 = 15,120)$  possible ways to create a two pair hand using this method.

Method 3: One of the two pairs of same-rank cards is of one of the ranks "A", "3", "6", "8", or "10", the other pair is of an unoccupied rank. The one remaining card is not of the ranks "A", "3", "6", "8", or "10":

1. For the first pair of same-rank cards, choose one of the ranks "A", "3", "6", "8", or "10". 5 possible choices.

2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the second pair of same-rank cards, choose a rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

4. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

5. For the one remaining card, choose a rank aside from "A", "3", "6", "8", "10", or the rank chosen in step 3. 7 possible choices.

6. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(5 * 3 * 8 * 6 * 7 * 4 = 20,160)$  possible ways to create a two pair hand using this method.

Method 4: One of the two pairs of same-rank cards is of one of the ranks "A", "3", "6", "8", or "10", the other pair is of an unoccupied rank. The one remaining card is of one of the unselected ranks of "A", "3", "6", "8", or "10":

1. For the first pair of same-rank cards, choose one of the ranks "A", "3", "6", "8", or "10". 5 possible choices.

2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. For the second pair of same-rank cards, choose a rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

4. Choose two of four suits for the second pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

5. For the one remaining card, choose one of the ranks "A", "3", "6", "8" and "10", that was not selected in step 1. 4 possible choices.

6. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(5 * 3 * 8 * 6 * 4 * 3 = 8,640)$  possible ways to create a two pair hand using this method.

Method 5: Both pairs of same-rank cards are of the ranks "A", "3", "6", "8", and "10". The one remaining card is not of the ranks "A", "3", "6", "8",

or “10”:

1. For the two pairs of same-rank cards, choose any two ranks of “A”, “3”, “6”, “8”, and “10”. Order does not matter.  $(C(5, 2) = 10)$  possible choices.

2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. Choose two of three suits for the second pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

4. For the one remaining card, choose a rank aside from “A”, “3”, “6”, “8”, or “10”. 8 possible choices.

5. Choose one of four suits for the one remaining card. 4 possible choices.

By the rule of multiplication, there are  $(10 * 3 * 3 * 8 * 4 = 2,880)$  possible ways to create a two pair hand using this method.

Method 6: Both pairs of same-rank cards are of the ranks “A”, “3”, “6”, “8”, and “10”. The one remaining card is of one of the unselected ranks of “A”, “3”, “6”, “8”, and “10”:

1. For the two pairs of same-rank cards, choose any two ranks of “A”, “3”, “6”, “8”, and “10”. Order does not matter.  $(C(5, 2) = 10)$  possible choices.

2. Choose two of three suits for the first pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

3. Choose two of three suits for the second pair of same-rank cards.  $(C(3, 2) = 3)$  possible choices.

4. For the one remaining card, choose one of the ranks “A”, “3”, “6”, “8”, and “10” that was not selected in step 1. 3 possible choices.

5. Choose one of three suits for the one remaining card. 3 possible choices.

By the rule of multiplication, there are  $(10 * 3 * 3 * 3 * 3 = 810)$  possible ways to create a two pair hand using this method.

Using the rule of addition, we see that there are  $(24,192 + 15,120 + 20,160 + 8,640 + 2,880 + 810 = 71,802)$  possible ways for your opponent to form a two pair hand.

A one pair hand can beat yours. There are 8 methods that your opponent can use to form a one pair hand.

Method 1: The pair of same-rank hands is not of the ranks “A”, “3”, “6”, “8”, or “10”. None of the three remaining cards are of the ranks “A”, “3”, “6”, “8”, or “10”:

1. For the pair of same-rank hands, choose a rank aside from “A”, “3”, “6”, “8”, or “10”. 8 possible choices.

2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. For the three remaining cards, choose three ranks aside from “A”, “3”, “6”, “8”, “10”, and the rank chosen in step 1. Order does not matter.  $(C(7, 3) = 35)$  possible choices.

4. Choose one of four suits for the first remaining card. 4 possible choices.

5. Choose one of four suits for the second remaining card. 4 possible choices.

6. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(8 * 6 * 35 * 4 * 4 * 4 = 107,520)$  possible ways to create a one pair hand using this method.

Method 2: The pair of same-rank hands is not of the ranks "A", "3", "6", "8", and "10". One of the three remaining cards is of one of the ranks "A", "3", "6", "8", and "10". The other two remaining cards are not of the ranks "A", "3", "6", "8", and "10":

1. For the pair of same-rank hands, choose a rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. For the first remaining card, choose one of the ranks "A", "3", "6", "8" and "10". 5 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. For the other two remaining cards, choose two ranks aside from "A", "3", "6", "8", "10", and the rank chosen in step 1. Order does not matter.  $(C(7, 2) = 21)$  possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(8 * 6 * 5 * 3 * 21 * 4 * 4 = 241,920)$  possible ways to create a one pair hand using this method.

Method 3: The pair of same-rank hands is not of the ranks "A", "3", "6", "8", or "10". Two of the three remaining cards are of the ranks "A", "3", "6", "8", and "10". The other one remaining card is not of the ranks "A", "3", "6", "8", and "10":

1. For the pair of same-rank hands, choose a rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

2. Choose two of four suits for the pair of same-rank cards.  $(C(4, 2) = 6)$  possible choices.

3. For the first two remaining cards, choose any two ranks of "A", "3", "6", "8", and "10". Order does not matter.  $(C(5, 2) = 10)$  possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

6. For the third remaining card, choose any rank aside from "A", "3", "6", "8", "10", or the rank chosen in step 1. 7 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(8 * 6 * 10 * 3 * 3 * 7 * 4 = 120,960)$  possible ways to create a one pair hand using this method.

Method 4: The pair of same-rank hands is not of the ranks "A", "3", "6", "8", or "10". The three remaining cards are all of the ranks "A", "3", "6", "8", and "10".

1. For the pair of same-rank hands, choose a rank aside from "A", "3", "6", "8", or "10". 8 possible choices.

2. Choose two of four suits for the pair of same-rank cards. ( $C(4, 2) = 6$ ) possible choices.

3. For the three remaining cards, choose any three ranks of "A", "3", "6", "8", and "10". Order does not matter. ( $C(5, 3) = 10$ ) possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

6. Choose one of three suits for the third remaining card. 3 possible choices.

By the rule of multiplication, there are  $(8 * 6 * 10 * 3 * 3 * 3 = 12,960)$  possible ways to create a one pair hand using this method.

Method 5: The pair of same-rank hands is of one of the ranks "A", "3", "6", "8", and "10". None of the three remaining cards are of the ranks "A", "3", "6", "8", and "10".

1. For the pair of same-rank cards, choose one of the ranks "A", "3", "6", "8", and "10". 5 possible choices.

2. Choose two of three suits for the pair of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

3. For the three remaining cards, choose any three ranks aside from "A", "3", "6", "8", and "10". Order does not matter. ( $C(8, 3) = 56$ ) possible choices.

4. Choose one of four ranks for the first remaining card. 4 possible choices.

5. Choose one of four ranks for the second remaining card. 4 possible choices.

6. Choose one of four ranks for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(5 * 3 * 56 * 4 * 4 * 4 = 53,760)$  possible ways to create a one pair hand using this method.

Method 6: The pair of same-rank hands is of one of the ranks "A", "3", "6", "8", and "10". One of the three remaining cards is of one of the unselected ranks of "A", "3", "6", "8", and "10". The other two remaining cards are not of the ranks "A", "3", "6", "8", and "10":

1. For the pair of same-rank cards, choose one of the ranks "A", "3", "6", "8", and "10". 5 possible choices.

2. Choose two of three suits for the pair of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

3. For the first remaining card, choose one of the ranks "A", "3", "6", "8", and "10" that was not selected in step 1. 4 possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. For the other two remaining cards, choose any two ranks aside from "A", "3", "6", "8", and "10". Order does not matter. ( $C(8, 2) = 28$ ) possible choices.

6. Choose one of four suits for the second remaining card. 4 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(5 * 3 * 4 * 3 * 28 * 4 * 4 = 80,640)$  possible ways to create a one pair hand using this method.

Method 7: The pair of same-rank hands is of one of the ranks "A", "3", "6", "8", and "10". Two of the three remaining cards are of the unselected ranks of

“A”, “3”, “6”, “8”, and “10”. The other one remaining card is not of the ranks “A”, “3”, “6”, “8”, and “10”:

1. For the pair of same-rank cards, choose one of the ranks “A”, “3”, “6”, “8”, and “10”. 5 possible choices.

2. Choose two of three suits for the pair of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

3. For the first two remaining cards, choose any two ranks of “A”, “3”, “6”, “8”, and “10” that were not selected in step 1. ( $C(4, 2) = 6$ ) possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

6. For the third remaining card, choose any rank aside from “A”, “3”, “6”, “8”, and “10”. 8 possible choices.

7. Choose one of four suits for the third remaining card. 4 possible choices.

By the rule of multiplication, there are  $(5 * 3 * 6 * 3 * 3 * 8 * 4 = 25,920)$  possible ways to create a one pair hand using this method.

Method 8: The pair of same-rank hands is of one of the ranks “A”, “3”, “6”, “8”, and “10”. The three remaining cards are all in the unselected ranks of “A”, “3”, “6”, “8”, and “10”.

1. For the pair of same-rank cards, choose one of the ranks “A”, “3”, “6”, “8”, and “10”. 5 possible choices.

2. Choose two of three suits for the pair of same-rank cards. ( $C(3, 2) = 3$ ) possible choices.

3. For the three remaining cards, choose any three ranks of “A”, “3”, “6”, “8”, and “10” that were not selected in step 1. ( $C(4, 3) = 4$ ) possible choices.

4. Choose one of three suits for the first remaining card. 3 possible choices.

5. Choose one of three suits for the second remaining card. 3 possible choices.

6. Choose one of three suits for the third remaining card. 3 possible choices.

By the rule of multiplication, there are  $(5 * 3 * 4 * 3 * 3 * 3 = 1,620)$  possible ways to create a one pair hand using this method.

Using the rule of addition, we can see that there are  $(107,520 + 241,920 + 120,960 + 12,960 + 53,760 + 80,640 + 25,920 + 1,620 = 645,300)$  possible ways for your opponent to form a one pair hand.

Number of hands that can tie with yours:

A no pair hand (a hand that does not fall under any of the hand types above) can tie with yours. We can find the number of possible no pair hands by simply subtracting the number of any non-no pair hand from the total number of possible hands for your opponent. There are 1,533,939 total possible hands, of which 759,695 are non-no pair hands. As a result, there are  $(1,533,939 - 759,695 = 774,244)$  no pair hands.

No pair probability tables:

Number of hands that beat yours	Chance of being beaten
Royal Flush: 2 Straight Flush: 21 Four of a kind: 344 Full house: 2,124 Flush: 2,815 Straight: 5,785 Three of a kind: 31,502 Two pair: 71,802 One pair: 645,300 Total: 759,695	$\frac{759,695}{1,533,939} = 49.53\%$
Number of hands that tie with yours	Chance of being tied
No pair: 774,244	$\frac{774,244}{1,533,939} = 50.47\%$
Chance of winning	
$100\% - 49.53\% - 50.47\% = 0\%$	

### 3.11 Table of Probabilities

Here are the chances of winning, losing, and tying associated with each poker hand.

Hand Type	Chance to Win	Chance to Tie	Chance to Lose
Royal flush	99.9998%	0.00020%	0%
Straight flush	99.998%	0.0018%	0.00020%
Four of a kind	99.9953%	0.0031%	0.0016%
Full house	99.798%	0.17%	0.032%
Flush	99.59%	0.25%	0.16%
Straight	99.25%	0.40%	0.35%
Three of a kind	96.96%	2.31%	0.73%
Two pair	92.82%	4.46%	2.72%
One pair	35.72%	56.58%	7.70%
No pair	0%	50.47%	49.53%

One thing that should be mentioned is that ties are rare in actual poker

games because if two players have the same hand type, then the tie is broken based on the rank of cards present in the hand, if we assume that whenever you “tie” you win half the time, and lose the other half of the time, what are the probabilities of winning and losing with each hand? We will consider royal flush ties as “wins” because all 4 royal flush hands have equal “value”, and will always tie with each other.

Hand Type	Chance to Win	Chance to Lose
Royal flush	100%	0%
Straight flush	99.9989%	0.0011%
Four of a kind	99.99685%	0.00315%
Full house	99.883%	0.117%
Flush	99.715%	0.285%
Straight	99.45%	0.55%
Three of a kind	98.115%	1.885%
Two pair	95.05%	4.95%
One pair	64.01%	35.99%
No pair	25.235%	74.765%

What if we multiply the chance of getting each hand with the probability of winning with each hand?

Hand Type	Chance to Win
Royal flush	$100\% * 0.00015\% = 0.00015\%$
Straight flush	$99.9989\% * 0.0014\% = 0.0014\%$
Four of a kind	$99.99685\% * 0.024\% = 0.024\%$
Full house	$99.883\% * 0.14\% = 0.14\%$
Flush	$99.715\% * 0.20\% = 0.20\%$
Straight	$99.45\% * 0.39\% = 0.39\%$
Three of a kind	$98.115\% * 2.11\% = 2.07\%$
Two pair	$95.05\% * 4.75\% = 4.51\%$
One pair	$64.01\% * 42.26\% = 27.05\%$
No pair	$25.235\% * 50.12\% = 12.65\%$
Cumulative probability of all hands	47.04%

Adding up the probabilities of getting each poker hand multiplied by the probability of winning with that specific poker hand, we see that you have an

approximately 47.04% chance of winning any given poker game against one opponent.

If you are facing off against two opponents, then your hand has to be better than both of their hands in order for you to win. As a result, if  $P$  is the probability of a given poker hand winning against one opponent, then the probability of that same poker hand winning against  $k$  opponents should be  $P^k$ .

## 4 Optimal Betting Strategy

Suppose that a game of Poker requires you to first enter \$10 into a betting pot to play. After the initial forced bet, all players are dealt a hand of 5 random cards.

Once every player is dealt a hand, each player takes turns either choosing to "raise" and stay in the game by putting in an amount of money into the pot that is greater than or equal to the amount of money bet by the last player who raised, or choosing to "fold" which does not cost any additional money, but prevents you from participating in the round further and you lose your forced initial bet. We will assume that you have no opportunities to replace any cards in your hand after you are dealt your 5 initial cards.

After every player has chosen either to raise or fold, every player who has not folded reveals their cards. The player with the highest ranking hand wins.

With the probability tables at the end of section 3, one can get a good idea of whether they should raise or fold given knowledge of what hand they possess. If you are playing against more than one opponent, then you can find the probability of winning against  $k$  players by raising the probability  $P$  to the  $k$ th power, where  $k$  is the number of opponents you are playing against.

If you plan on playing only once, then you should probably raise only if there is a 50% chance or more that your hand will beat or at least tie with any other random hand.

If you plan on playing multiple games, then you can use a CMF to find the cumulative probability of winning at least  $x$  out of  $y$  games. If you are playing against only one opponent, then we have already found that the probability of winning any given game is approximately 47.04%. If you are playing against  $k$  opponents, then you can find the probability of winning any given game by first raising the probability of winning with each hand to the  $k$ th power, then summing up the probabilities of getting each hand multiplied by the probability of winning with that hand.

I think playing  $y$  games should only be considered profitable if the CMF of winning at least as many as half of the games is greater than or equal to 50%.