Applications of Bayes' Theorem

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Thomas Bayes

- 1701-1761
- English minister, philosopher, and statistician
- Son of a minister
- Enrolled at University of Edinburgh
- Fellow of the Royal Society
- Bayes' theorem

Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(\bar{A})P(B|\bar{A})}$$

- This theorem is used to find a probability when other probabilities are known.
- It's also about relating how probabilities change when knowledge increases.

Example 1

Let's say P(r) means how often rain occurs, and P(c) means how often clouds are seen. We can then say P(r|c) denotes how often rain occurs when clouds can be seen, and P(c|r) denotes how often clouds can be seen when there is rain.

We also know that dangerous rainstorms have a 1% chance of occurring, clouds are common (10%), and 80% of dangerous rainstorms are preceded by clouds.

Let's calculate the probability of a dangerous rainstorm occurring when clouds can be seen.

$$P(r|c) = \frac{P(r)P(c|r)}{P(c)}$$

$$= \frac{(0.01)(0.8)}{0.1}$$

$$= 8\%$$

Example 2

Let's take a drawer that contains two black socks and one white sock. Another drawer contains one black sock and three white socks.

Let's toss a coin. The drawer from which we randomly select a sock is determined by the flip.

Find the probability that the sock chosen came from the first drawer if the sock is black.

$$Let\ A{:}Drawer\ 1, \bar{A}{:}\ Drawer\ 2. Thus,$$

$$P(A) = P(\bar{A}) = \frac{1}{2}.$$

$$Let\ B{:}Black\ sock, \bar{B}{:} White\ sock. Thus,$$

$$P(B|A) = \frac{2}{3}, P(\bar{B}|A) = \frac{1}{3},$$

$$and\ P(B|\bar{A}) = \frac{1}{4}, P(\bar{B}|\bar{A}) = \frac{3}{4}.$$

$$P(A|B) = \frac{P(A)P(B|A)}{P(B|A)P(A) + P(\bar{A})P(B|\bar{A})}$$

$$= \frac{8}{11}$$

Financial Statement Auditing

The truth of some things is not known, but we must reach a conclusion. So, we need multiple sources to reach a valid opinion.

Keep in mind:

- Auditors can be sued if there is a material misstatement in financial statements that he renders an opinion on.
- Auditors can't test every transaction, so can never know the true extent of errors.
- Auditors use a variety of techniques to gather information to support opinions.

Bayes' Theorem in Audit

Auditors can't test every transaction, so can never know the true extent of errors. Instead, they consider the results of each audit procedure to determine the potential impact on other accounts and determine the level of risk of error.

Bayes' Theorem is used in determining required thresholds of testing in:

- Lead sheet analytics
- Journal entry testing for fraud
- Cost of sales testing to provide evidence of inventory

Machine Learning

- UPS uses AI to prevent ‘porch pirates’ from stealing.
- Travel websites recommend places to visit and resorts to stay.
- Search engines suggest the phrases a user will most likely want.
- We can perform Bayesian inference over mixture models.

Thank You!