

1. What is the largest interval on which a solution to the IVP is guaranteed to exist?

a. $(\sin t)y' - t^2y = \frac{t}{t^2 - 16}, \quad y(1) = 2\pi$

b. $(\sin t)y' - t^2y = \frac{t}{t^2 - 16}, \quad y(3.5) = -1$

2. a. Verify that both $y_1(t) = 1 - t$ and $y_2(t) = -t^2/4$ are solutions to the IVP:

$$y' = \frac{-t + (t^2 + 4y)^{1/2}}{2}, \quad y(2) = -1$$

b. Explain why the existence of two solutions through the same point does not contradict the uniqueness theorem.

3. Find and classify all equilibrium points, draw a phase line, and sketch solutions in the yt -plane.

a. $y' = y^2(1 - y)$

b. $y' = y \ln \left(y^2 + \frac{3}{4} \right)$