

For problems 1–3,

- a. Find all critical points.
- b. For each critical point, find the corresponding linear system.
- c. Find eigenvalues of each linear system.
- d. Draw a phase portrait for the nonlinear system. Note: It may be necessary to go beyond the work in part c to do this accurately.

1. $dx/dt = x - y^2$, $dy/dt = x - 2y + x^2$

2. $dx/dt = 3 - xy$, $dy/dt = x - 3y^3$

3. $dx/dt = 4 - y^2$, $dy/dt = (1.5 + x)(y - x)$

4. (*optional*) Consider the system: $dx/dt = -\sqrt{x^2 + y^2}$, $dy/dt = \sqrt{x^2 + y^2}$. There is a single critical point at the origin. Instead of analyzing this problem as above, try to reason about what should happen in this system.

- a. What does the quantity $\sqrt{x^2 + y^2}$ represent?
- b. If you were to draw a direction field, what would be the direction of any vector? What is the magnitude?
- c. Draw the phase portrait.