Implementation of Euler's Method

Example 8.1 from Brannon and Boyce

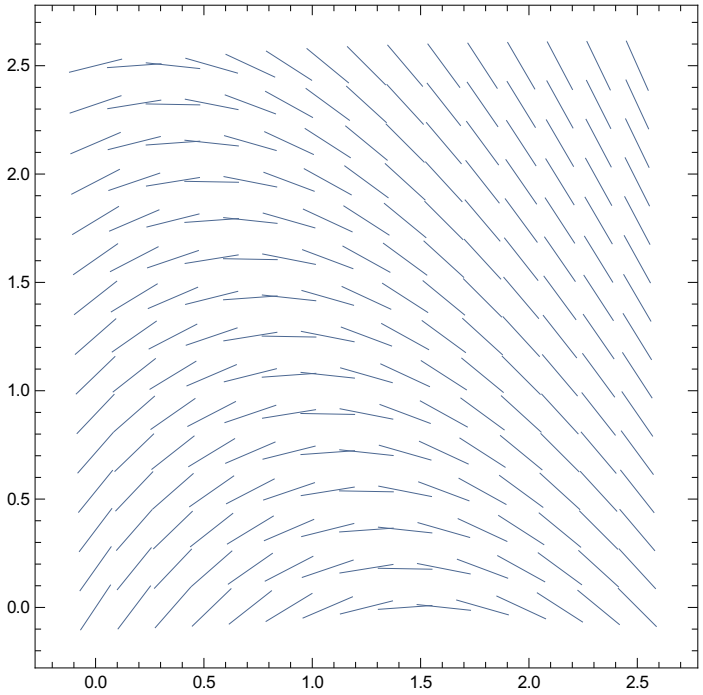
ODE: y' = -y/2 + 3/2 - t

In[1]:=
```
f[t_, y_] = -y / 2 + 3 / 2 - t
```

Out[1]=
$$\frac{3}{2} - t - \frac{y}{2}$$

In[3]:=
```
sf = VectorPlot[{1, f[t, y]} / Norm[{1, f[t, y]}],
  {t, 0, 2.5}, {y, 0, 2.5}, VectorStyle → Arrowheads[0]]
```

Out[3]=



Initial value: y(0) = 1
We will approximate on the interval [0,1] with stepsize h = 1/N

In[10]:=
```
tzero = 0;
yzero = 1;
```

In[4]:=
```
euler[N_] := For[j = 1; soln = {{tzero, yzero}}, j < N + 1, j++, soln = Union[soln,
    {{tzero + j (1 / N), soln[[j]][[2]] + f[tzero + (j - 1) / N, soln[[j]][[2]]] (1 / N)}}]]
```

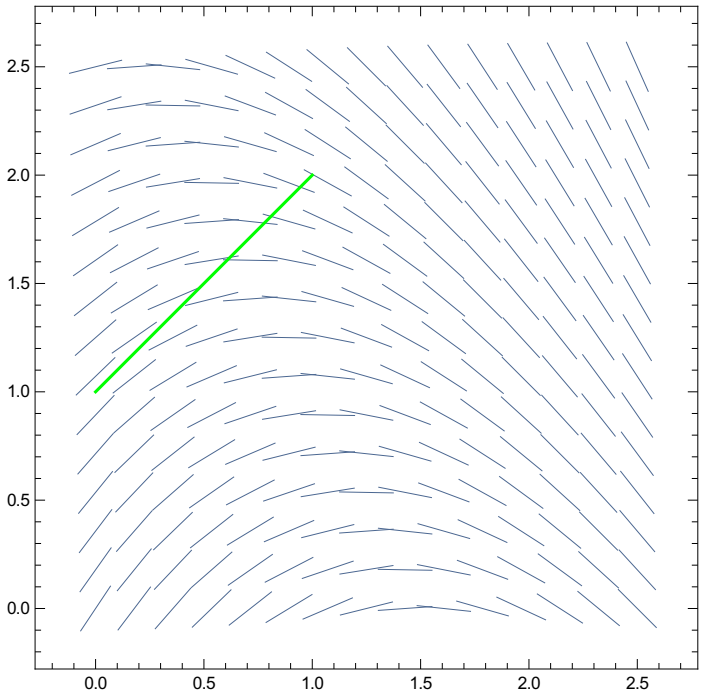
In[12]:=
```
euler[1]
```

In[13]:=
```
soln
```

Out[13]=
```
{{0, 1}, {1, 2}}
```

In[14]:=
```
show1 = Show[sf, ListPlot[soln, Joined → True, PlotStyle → Green]]
```

Out[14]=

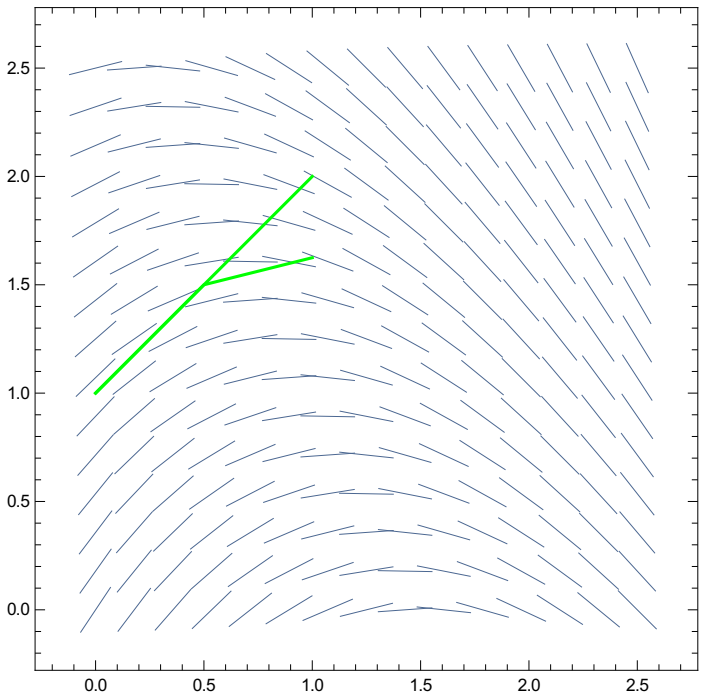


In[15]:=
```
euler[2]
soln
```

Out[16]=
$$\left\{\{0,\ 1\},\ \left\{\frac{1}{2},\ \frac{3}{2}\right\},\ \left\{1,\ \frac{13}{8}\right\}\right\}$$

In[17]:=
```
show2 = Show[show1, ListPlot[soln, Joined → True, PlotStyle → Green]]
```

Out[17]=



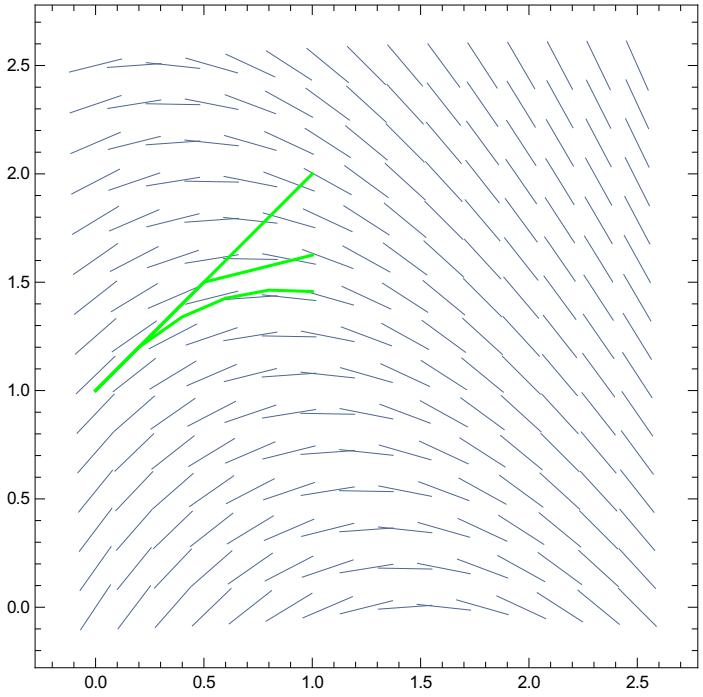
In[30]:=
```
euler[5]
soln
N[soln]
```

Out[31]= $\left\{\{0, 1\}, \left\{\frac{1}{5}, \frac{6}{5}\right\}, \left\{\frac{2}{5}, \frac{67}{50}\right\}, \left\{\frac{3}{5}, \frac{713}{500}\right\}, \left\{\frac{4}{5}, \frac{7317}{5000}\right\}, \left\{1, \frac{72\,853}{50\,000}\right\}\right\}$

Out[32]= {{0., 1.}, {0.2, 1.2}, {0.4, 1.34}, {0.6, 1.426}, {0.8, 1.4634}, {1., 1.45706}}

In[20]:=
```
show3 = Show[show2, ListPlot[soln, Joined → True, PlotStyle → Green]]
```

Out[20]=

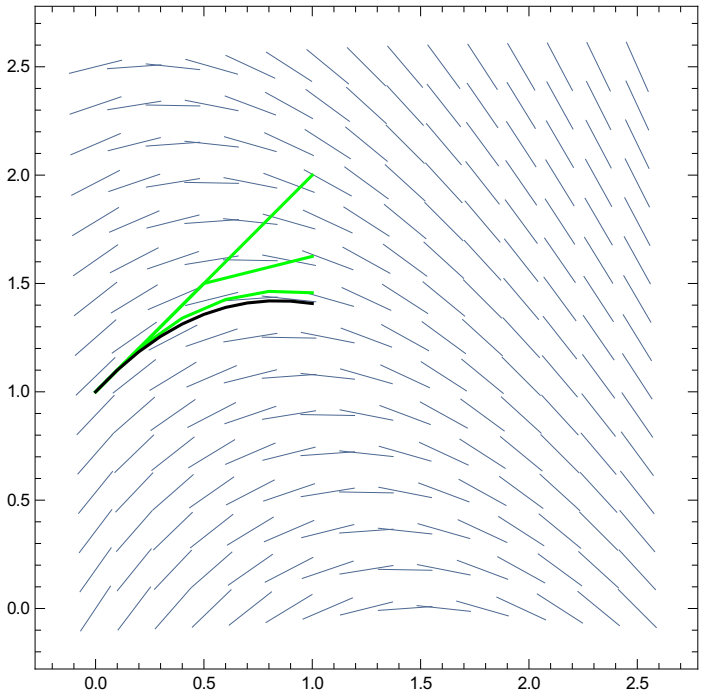


In[21]:=
```
euler[10]
soln
```

Out[22]= $\left\{\{0, 1\}, \left\{\frac{1}{10}, \frac{11}{10}\right\}, \left\{\frac{1}{5}, \frac{237}{200}\right\}, \left\{\frac{3}{10}, \frac{5023}{4000}\right\},\right.$
$\left\{\frac{2}{5}, \frac{105\,037}{80\,000}\right\}, \left\{\frac{1}{2}, \frac{2\,171\,703}{1\,600\,000}\right\}, \left\{\frac{3}{5}, \frac{44\,462\,357}{32\,000\,000}\right\}, \left\{\frac{7}{10}, \frac{902\,384\,783}{640\,000\,000}\right\},$
$\left.\left\{\frac{4}{5}, \frac{18\,169\,310\,877}{12\,800\,000\,000}\right\}, \left\{\frac{9}{10}, \frac{363\,136\,906\,663}{256\,000\,000\,000}\right\}, \left\{1, \frac{7\,206\,801\,226\,597}{5\,120\,000\,000\,000}\right\}\right\}$

In[24]:=
```
show4 = Show[show3, ListPlot[soln, Joined → True, PlotStyle → Black]]
```

Out[24]=



In[25]:=
```
nsoln = NDSolve[{ny'[nt] == f[nt, ny[nt]], ny[tzero] == yzero}, ny, {nt, 0, 1}]
```

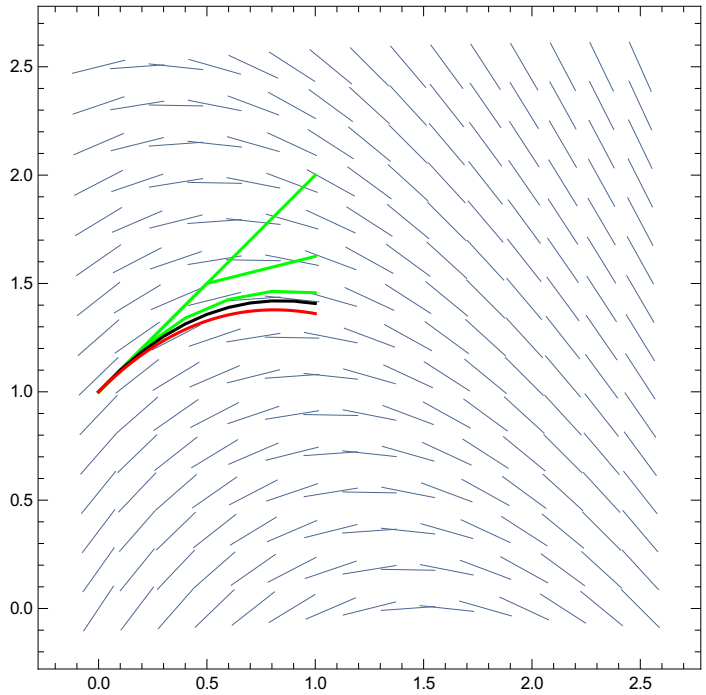
Out[25]=
```
{{ny → InterpolatingFunction[ ⊞ Domain: {{0., 1.}}
                              Output: scalar ]}}
```

In[27]:=
```
rky[t_] = ny[t] /. nsoln[[1]]
```

Out[27]=
```
InterpolatingFunction[ ⊞ Domain: {{0., 1.}}
                       Output: scalar ][t]
```

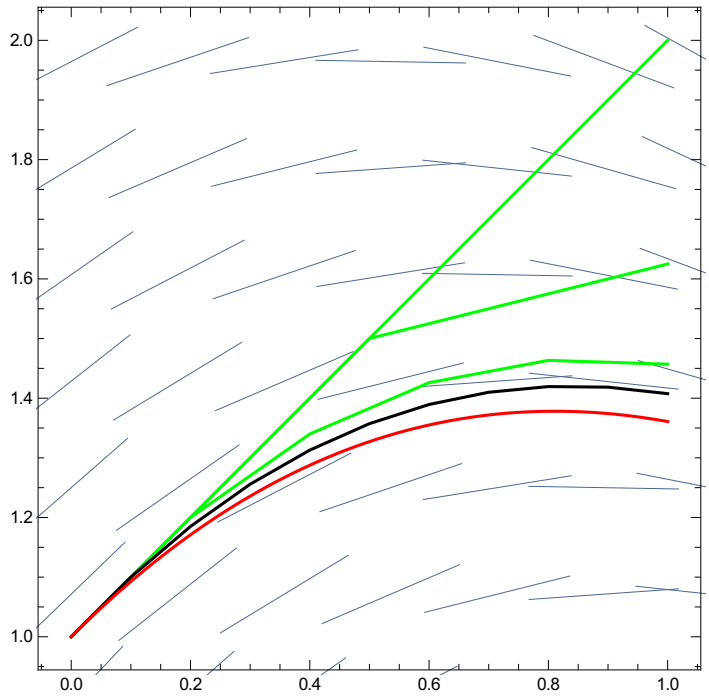
In[28]:=
```
Show[show4, Plot[rky[t], {t, 0, 1}, PlotStyle → Red]]
```

Out[28]=



In[29]:=
```
Show[show4, Plot[rky[t], {t, 0, 1}, PlotStyle → Red], PlotRange → {{0, 1}, {1, 2}}]
```

Out[29]=



Improved Euler Method

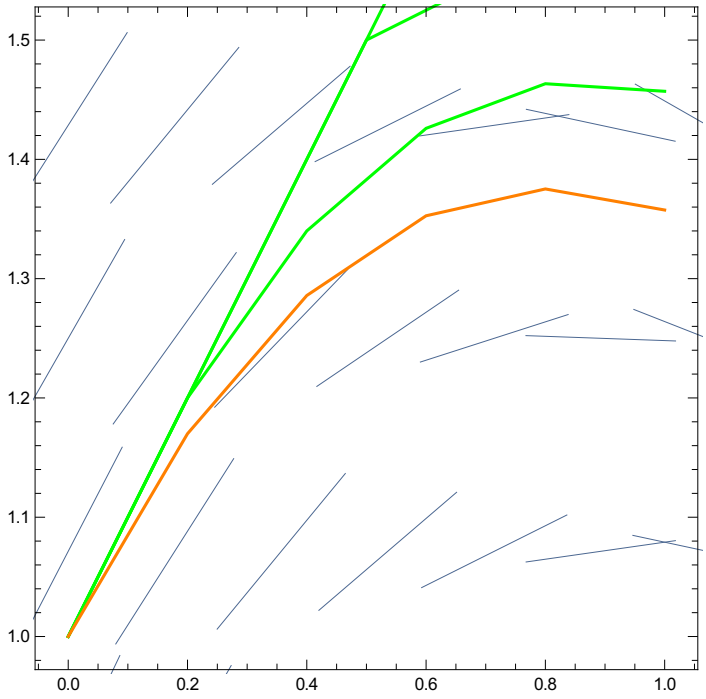
In[33]:=
```
ieuler[N_] := For[j = 1; soln = {{tzero, yzero}},
  j < N + 1, j ++, mone = f[tzero + (j - 1) / N, soln[[j]][[2]]];
  mtwo = f[tzero + j / N, soln[[j]][[2]] + mone (1 / N)];
  soln = Union[soln, {{tzero + j (1 / N), soln[[j]][[2]] + ((mone + mtwo) / 2) (1 / N)}}]]
```

In[40]:=
```
ieuler[5]
N[soln]
```

Out[41]=
```
{{0., 1.}, {0.2, 1.17}, {0.4, 1.28585}, {0.6, 1.35269}, {0.8, 1.37519}, {1., 1.35755}}
```

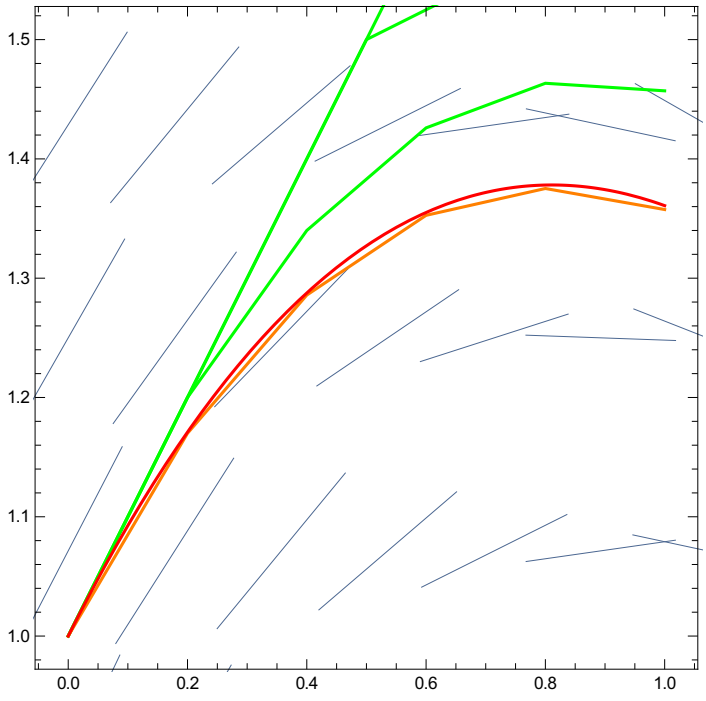
In[44]:=
```
Show[show3, ListPlot[soln, Joined → True, PlotStyle → Orange],
 PlotRange → {{0, 1}, {1, 1.5}}]
```

Out[44]=



In[43]:=
```
Show[show3, ListPlot[soln, Joined → True, PlotStyle → Orange],
 Plot[rky[t], {t, 0, 1}, PlotStyle → Red], PlotRange → {{0, 1}, {1, 1.5}}]
```

Out[43]=



Fourth Order Runge-Kutta Method

In[45]:=
```
rk[N_] := For[j = 1; soln = {{tzero, yzero}},
  j < N + 1, j++, mone = f[tzero + (j - 1) / N, soln[[j]][[2]]];
  mtwo = f[soln[[j]][[1]] + 1 / (2 N), soln[[j]][[2]] + mone (1 / (2 N))];
  mthree = f[soln[[j]][[1]] + 1 / (2 N), soln[[j]][[2]] + mtwo (1 / (2 N))] ;
  mfour = f[tzero + j / N, soln[[j]][[2]] + mthree (1 / N)];
  soln = Union[soln,
    {{tzero + j (1 / N), soln[[j]][[2]] + ((mone + 2 mtwo + 2 mthree + mfour) / 6) (1 / N)}}]]
```

In[46]:=
```
rk[5]
N[soln]
```

Out[47]=
```
{{0., 1.}, {0.2, 1.17098}, {0.4, 1.28761},
 {0.6, 1.35509}, {0.8, 1.37808}, {1., 1.36081}}
```

In[48]:=
```
Show[ListPlot[soln, Joined → True, PlotStyle → Black],
 Plot[rky[t], {t, 0, 1}, PlotStyle → Red], PlotRange → {{0, 1}, {1, 1.5}}]
```

Out[48]=