Problem 7.3.4: A nonlinear system

ODE:

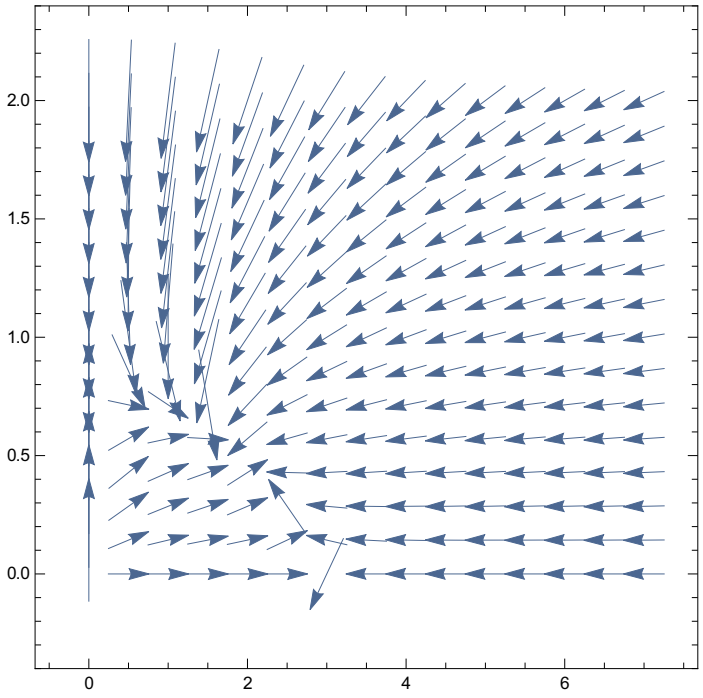
x' = x (1.5 - 0.5 x - y)
w' = y (0.75 - y -0.125 x)

In[70]:=
```
vf[x_, y_] = {x (1.5 - 0.5 x - y), y (0.75 - y - 0.125 x)}
```

Out[70]=
```
{x (1.5 - 0.5 x - y), (0.75 - 0.125 x - y) y}
```

In[71]:=
```
df = VectorPlot[vf[x, y] / Norm[vf[x, y]],
  {x, 0, 7}, {y, 0, 2}, VectorStyle → Arrowheads[.03]]
```

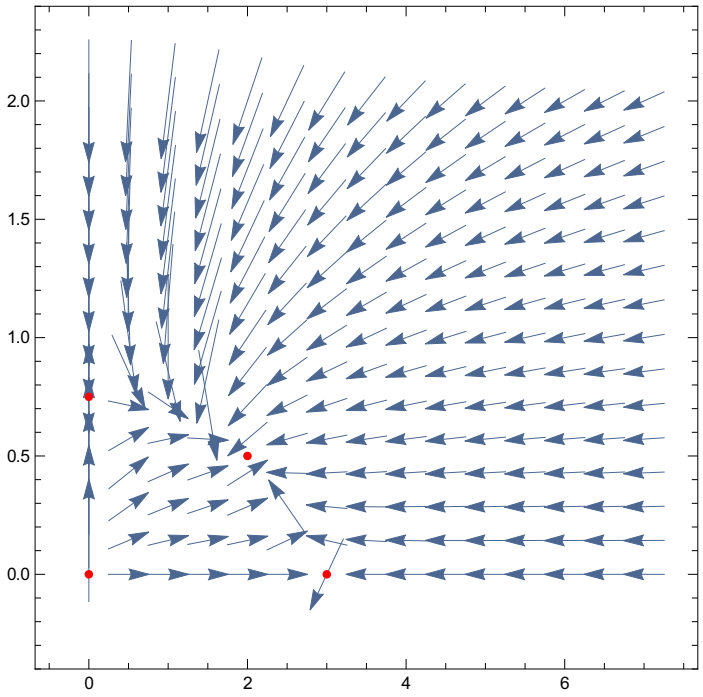
Out[71]=



Equilibria: (0,0), (3,0), (0,3/4), (2,1/2)

In[96]:=
```
dfeq = Show[df, ListPlot[{{0, 0}, {3, 0}, {0, 3 / 4}, {2, 1 / 2}}, PlotStyle → Red]]
```

Out[96]=



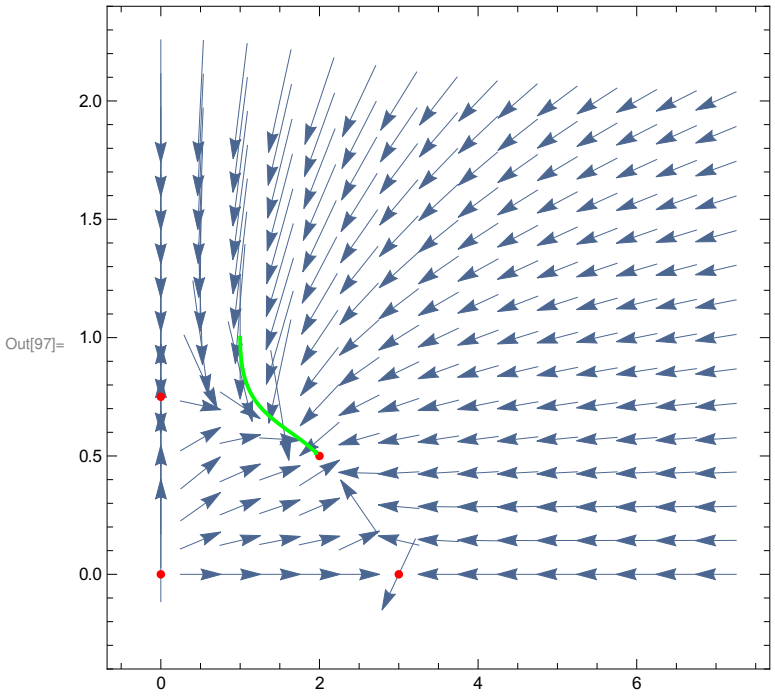
We know the behavior near the equilibrium points. Let's try to get some orbits.

In[98]:=
```
soln[xzero_?NumericQ, yzero_?NumericQ] := NDSolve[
  {dx'[t] == dx[t] (1.5 - 0.5 dx[t] - dy[t]), dy'[t] == dy[t] (0.75 - dy[t] - 0.125 dx[t]),
   dx[0] == xzero, dy[0] == yzero}, {dx, dy}, {t, 0, 10}]
xx[t_?NumericQ, xzero_?NumericQ, yzero_?NumericQ] :=
 dx[t] /. soln[xzero, yzero][[1]]
yy[t_?NumericQ, xzero_?NumericQ, yzero_?NumericQ] := dy[t] /. soln[xzero, yzero][[1]]
```

In[76]:=
```
orbit1 =
  ParametricPlot[{xx[s, 1, 1], yy[s, 1, 1]}, {s, 0, 10}, PlotStyle → {Thick, Green}];
```

In[97]:=
```
Show[dfeq, orbit1]
```

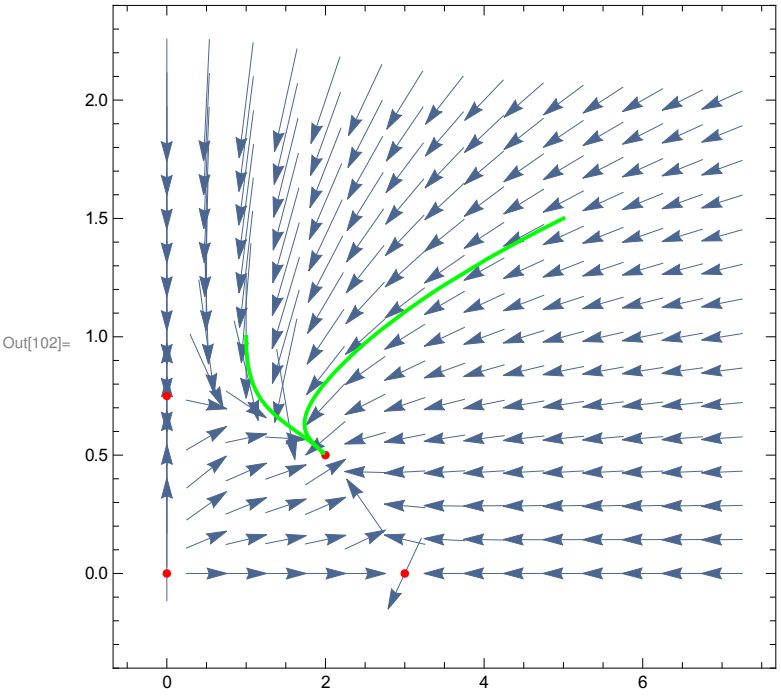
Out[97]=



In[101]:=
```
orbit2 = ParametricPlot[
    {xx[s, 5, 1.5], yy[s, 5, 1.5]}, {s, 0, 10}, PlotStyle → {Thick, Green}];
```

In[102]:=
```
Show[dfeq, orbit1, orbit2]
```

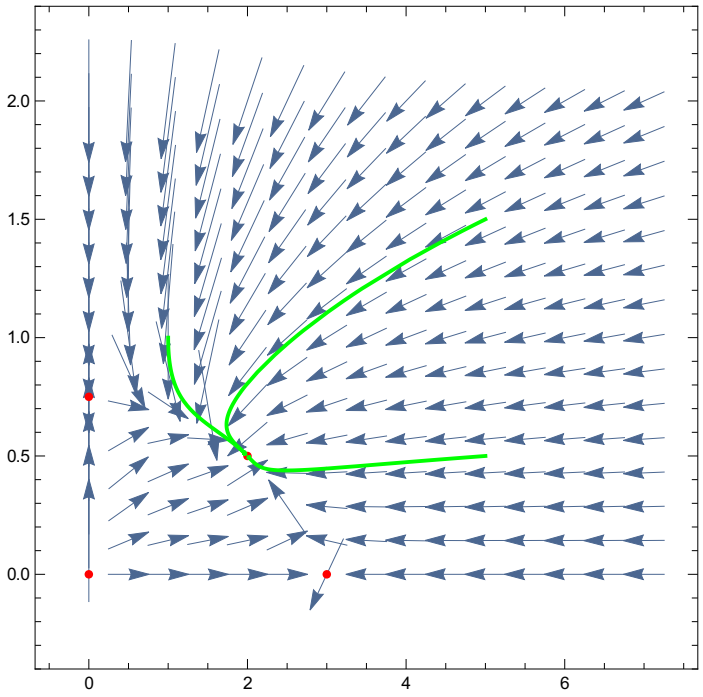
Out[102]=



In[103]:=
```
orbit3 = ParametricPlot[
    {xx[s, 5, 0.5], yy[s, 5, 0.5]}, {s, 0, 10}, PlotStyle → {Thick, Green}];
```

In[104]:=
```
Show[dfeq, orbit1, orbit2, orbit3]
```

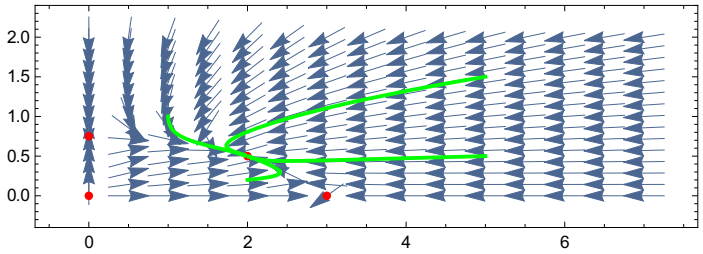
Out[104]=



In[106]:=
```
orbit4 = ParametricPlot[
    {xx[s, 2, 0.2], yy[s, 2, 0.2]}, {s, 0, 10}, PlotStyle → {Thick, Green}];
```

In[107]:=
```
Show[dfeq, orbit1, orbit2, orbit3, orbit4, AspectRatio → Automatic]
```

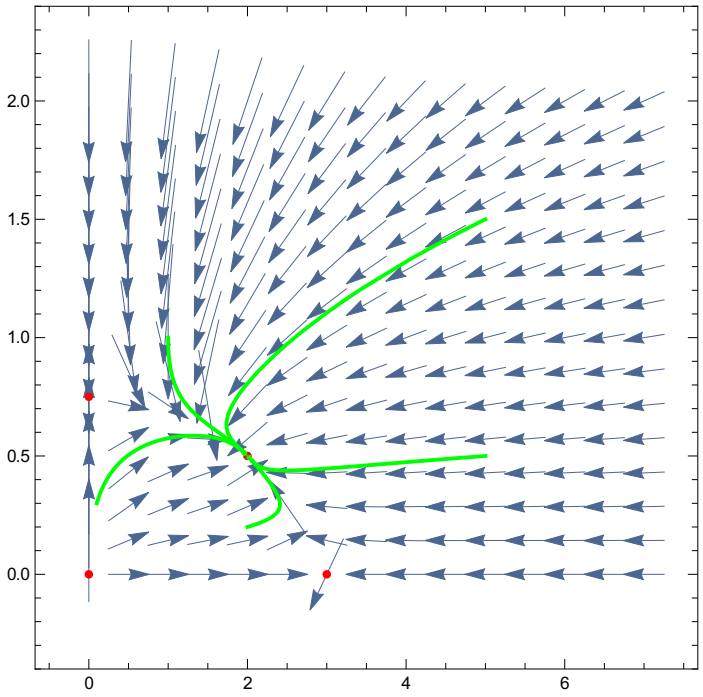
Out[107]=



In[108]:=
```
orbit5 = ParametricPlot[
    {xx[s, 0.1, 0.3], yy[s, 0.1, 0.3]}, {s, 0, 10}, PlotStyle → {Thick, Green}];
```

In[109]:=
```
Show[dfeq, orbit1, orbit2, orbit3, orbit4, orbit5]
```

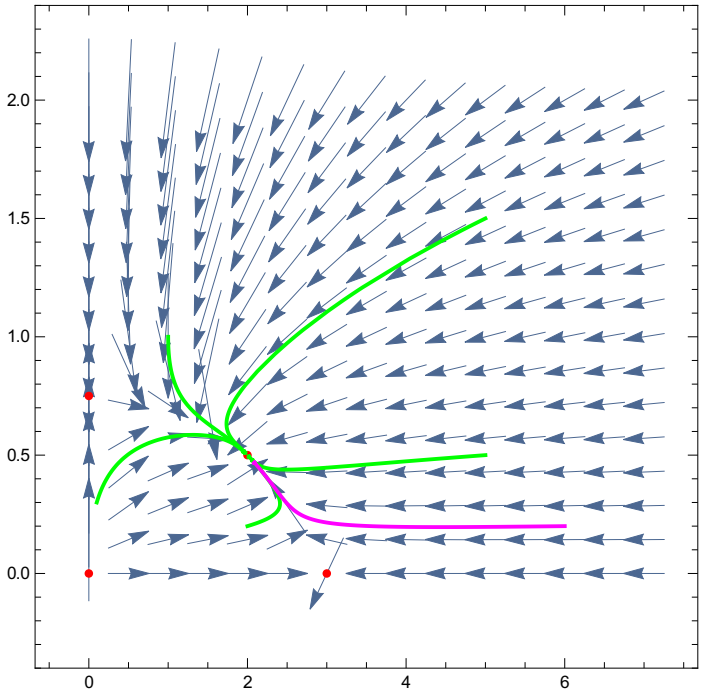
Out[109]=



In[110]:=
```
orbit6 = ParametricPlot[{xx[s, 6, 0.2], yy[s, 6, 0.2]},
    {s, 0, 10}, PlotStyle → {Thick, Magenta}];
```

In[111]:=
```
Show[dfeq, orbit1, orbit2, orbit3, orbit4, orbit5, orbit6]
```

Out[111]=



In[114]:=
```
orbit7 = ParametricPlot[{xx[s, 6, 0.02], yy[s, 6, 0.02]},
  {s, 0, 10}, PlotStyle → {Thick, Magenta}];
orbit8 = ParametricPlot[{xx[s, 0.5, 0.02], yy[s, 0.5, 0.02]},
  {s, 0, 10}, PlotStyle → {Thick, Magenta}]; orbit9 = ParametricPlot[
  {xx[s, 2, 0.02], yy[s, 2, 0.02]}, {s, 0, 10}, PlotStyle → {Thick, Magenta}];
```

In[115]:=
```
Show[dfeq, orbit1, orbit2, orbit3, orbit4, orbit5, orbit6, orbit7, orbit8, orbit9]
```

Out[115]=