Math 2550, Exam 2 form p Name and section: ____________________

1. Compute the partial derivatives

   (a) (15 points) (14.3.6) $f(x, y) = (2x - 3y)^3$

$$\frac{\partial f}{\partial x} = \qquad \text{and} \qquad \frac{\partial f}{\partial y} =$$

$$\frac{\partial^2 f}{\partial x^2} = \qquad , \frac{\partial^2 f}{\partial x \partial y} = \qquad \text{and} \qquad \frac{\partial^2 f}{\partial y^2} =$$

   (b) (10 points) (14.4.41) If $w = g(x^2 + y^3)$ and $g'(t) = e^t$, find

$$\frac{\partial w}{\partial x} = \qquad \text{and} \qquad \frac{\partial w}{\partial y} =$$

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2. (20 points) (14.5.15) Consider the function $f(x, y) = xyz$ on $\mathbb{R}^3$. Find the rate of change of f in the vector direction $\mathbf{v} = (2, 0, -4)$ at the point $\mathbf{p} = (2/3, 1, 4/3)$. This should be the same value as
$$\frac{d}{dt} f(\mathbf{r}(t))\Big|_{t=0} = D_{\mathbf{u}} f(\mathbf{p})$$
where $\mathbf{r}(t) = \mathbf{p} + t\mathbf{u}$ and $\mathbf{u} = \mathbf{v}/|\mathbf{v}|$ is a unit vector in the same direction as $\mathbf{v}$.

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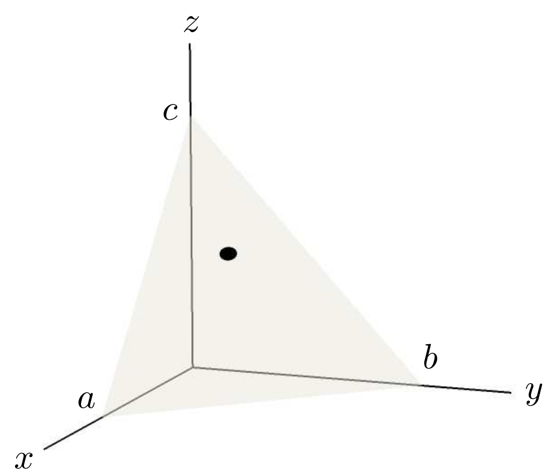
3. (14.6.7) Let a, b, and c be fixed positive numbers, and let $\mathcal{P}$ be the plane passing through $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$.



(a) (10 points) Find the equation of $\mathcal{P}$.

(b) (15 points) Give a parametric representation of the line normal to $\mathcal{P}$ passing through the point $(a/4, b/4, c/2)$.

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4. (25 points) (14.7.62) A rectangular solid (box) is formed with corners $(0, 0, 0)$, $(\ell, 0, 0)$, $(0, w, 0)$, $(0, 0, h)$, and (ℓ, w, h) with ℓ, w, and h all positive and (ℓ, w, h) on the plane

$$6x + 4y + 3z = 12.$$

Find the values of ℓ, w and h giving the box of largest volume.

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5. (10 points) (Bonus) Determine the maximizing dimensions of a box $[0, \ell] \times [0, w] \times [0, h]$ as in problem 4 with corner (ℓ, w, h) on the plane $\mathcal{P}$ determined by $(a, 0, 0)$, $(0, b, 0)$, and $(0, 0, c)$ from problem 2. Your answer should be in terms of a, b, and c.