

Midterm Exam 1 Practice, Math 2413

NAME: _____

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1. Find the general solution of the equation $x' = .04x(1000 - x)$.

2. Solve the IVP

$$\begin{cases} y' = \sin(x)/y \\ y(\pi/2) = 1. \end{cases}$$

3. If a radioactive element decays according to the ODE $N' = -\lambda N$, find a formula for the half-life of this element.

4. Find the Euler-Lagrange equation associated with the functional

$$\mathfrak{F}[y] = \int_0^1 \sin(y') \, dx.$$

5. Find the possible local minimizers of $\mathfrak{F}[y] = \int_0^\pi (y^2 - y'^2) \, dx$ on $\mathcal{A} = \{y \in C^1[0, \pi] : y(0) = y(\pi) = 0\}$.

6. A ball rolls atop an ellipsoid with long axis 8 inches and parallel to the ground and short axes both 2 inches. If the ball always rolls in the direction of quickest descent (i.e., ignore inertia), describe the path followed by the ball.