Assignment 6
Math 2413

October 31, 2008

1. Read bogus §4.3.

2. Look at the bogus problems 3.3.9,11,12,13; 3.4.1,4,17,20,22; 4.3.37,38; 4.4.19,20,22,23.

3. Find the Euler-Lagrange PDE associated with Dirichlet energy:

$$\mathcal{E}[u] = \int_\Omega |Du|^2.$$

   Go through the entire derivation.

4. Expand the mean curvature operator:

$$\operatorname{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = \frac{(1+u_y^2)u_{xx} - 2u_x u_y u_{xy} + (1+u_x^2)u_{yy}}{(1+|Du|^2)^{3/2}}.$$

   Can you identify the sum of the normal curvatures in this expression?

5. Read bogus §4.5

6. Look at the bogus problems 4.5.18,20,22,36,37,38.

7. Prove that if $\dot{\mathbf{x}} = \mathbb{F}(\mathbf{x})$ defines a dynamical system on $\mathbb{R}^2$, and $\mathbf{x}_* = (0,0)$ is an equilibrium point such that $D\mathbb{F}(\mathbf{x}_*)$ has two strictly negative eigenvalues, then $\mathbf{x}_*$ is asymptotically stable.

8. Solve the hanging chain ODE two different ways (by using the first integral and directly by the standard change of variables in a second order autonomous ODE).

9. Read bogus §4.7

10. Look at bogus problems 4.7.1,7,8,10,30.

11. Look at bogus Project 4.8 on Duffings equation.

12. Vocabulary: Harmonic Oscillator, Hooke's Law, RC circuit, RLC circuit, forcing, damping, resonance, flow.